

WINTER 2018

A MAGAZINE FOR MATHEMATICS AND SCIENCE EDUCATORS



TERC

AND



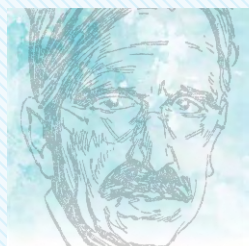
03

SportsLab: Opening the Door to 21st Century Careers
through a Sports Research and Shoe Design Challenge

CONTENTS

03

SportsLab:
Opening the Door to 21st
Century Careers through a
Sports Research and Shoe
Design Challenge



08

**A Durable Feast: Notes
from the TERC John Dewey
Group After 15 Years**

12

**Building Solutions for
Algebra Readiness**



16

Doing the Math with Paraeducators



Letter from the President

In this issue of Hands On! you will read about three exciting research projects. Although quite different in their content focus and approach, there is at least one common theme—allowing time for and supporting discussion, reflection and argumentation during an educational experience. This essential element in the learning process helps to build not only knowledge, but confidence and a strong sense of agency in the learner.

SportsLab is a project-based STEM learning experience which takes place in a collaborative game-like environment online. It leverages students' interest in sports to demonstrate the importance of STEM education while revealing related career opportunities. Forming small teams, students model real world product development activities using math and science concepts to research and design a Parkour shoe prototype.

The work of John Dewey has guided a number of TERC staff and colleagues over many years. In “Notes from the TERC John Dewey Group After 15 Years”, you will hear how this philosopher’s commitment to democracy as a process of daily life and his valuing of philosophy as a tool for understanding and improving social and individual growth has influenced four scientists’ body of work and life.

Our early algebra curriculum Project LEAP, built on a decade of research, is being piloted in schools across the country. We are examining its impact on children’s algebra understanding and their algebra readiness for middle grades. Experimental studies are showing positive and exciting results when students are taught the LEAP curriculum.

Paraeducators are asked every day to support students with different learning styles and needs in elementary classrooms. “Doing the Math with Paraeducators” introduces our professional development model, which aims to increase grades K-3 paraeducators’ confidence, knowledge of elementary math ideas, pedagogical strategies and understanding of how students learn mathematics.

Finally, to honor our deep roots in education, we are developing a historical timeline to capture the landmark moments, projects, and people that have made us who we are today and that are the foundation for our work ahead. We are starting with Sue Doubler.

Enjoy the issue.

Laurie

Laurie Brennan, *President*

Editors: Valerie Martin, Jenni Haley and Amanda Scott

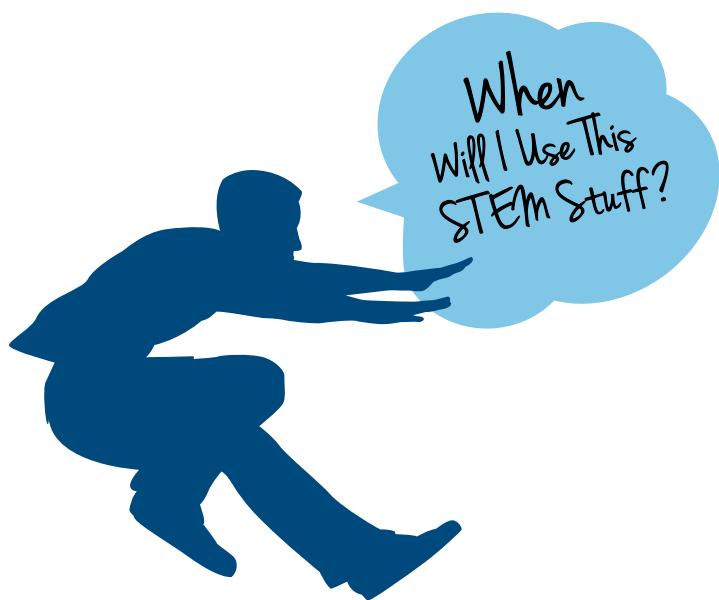
Administrative Support: Katie Stokinger

Design: One Visual Mind

SPORTSLAB

Opening the Door to 21st Century Careers through a Sports Research and Shoe Design Challenge

We've all known students in class who think they are destined for a career in professional sports. Too often kids with their eyes on a pro career have parents who feed a goal that is a long shot at best.



A recent NPR story cites a poll in which 26% of parents with sports-minded kids in high school hope their child will turn pro.¹ This number leaps to 39% in families with incomes below \$50,000. Both kids and parents hold tight to this dream, even though the NCAA reports the increasingly slim chances of any student playing at the college or professional level.

Of the nearly 8 million students currently participating in high school athletics in the United States, only 480,000 of them will compete at NCAA schools. And of that group, only a fraction will realize their goal of becoming a professional or Olympic athlete.²

Trying to get sports-minded kids excited about Science, Technology, Engineering and Mathematics (STEM) often elicits a rolling of eyes and a statement such as, "I don't need to worry about this science stuff, I'm going to be the next LeBron James!" Parents sometimes echo these sentiments: "My daughter doesn't have time for homework. She is on a traveling team and has practice before and after school and at tournaments every weekend." Citing the odds against a professional sports career is usually a losing argument and actually undercuts the value of sports and competition for instilling vital connections between educators and learners. It also ignores the positive lessons gained from pursuing an activity students are passionate about, lessons that can teach positive life-long learning skills—leadership, cooperation, resilience, and perseverance.

The sport-focused kid provides an extreme example of the disconnect we hear from many an unmotivated STEM student. They are not the only ones to pose the question we all have to be able to answer: *When am I going to use this stuff?* In this respect, an important part of our job as educators is to connect our teaching to interesting real-world careers, especially careers our students might never learn about otherwise.

26% of parents with sports-minded kids in high school hope their child will turn pro.



SPORTSLAB Opening the Door to 21st Century Careers



Figure 1. SportsLab's environment offers many opportunities to explore STEM careers connected to sport research and product design by visiting various locations like a sport research lab, Parkour Gym, and makerspace.

While many kids don't see how STEM might be connected to their lives, many sport product companies, as well as other businesses, wonder where they will find the STEM-trained workers they need to fill their immediate and future job needs. The Smithsonian Science Education Center claims that currently (2018), as many as 2.4 million of these jobs go unfilled.³ Minorities are hugely underrepresented in STEM careers, as just 2.2% of Latinos, 2.7% of African Americans, and 3.3% of Native Americans and Alaska Natives earn a degree in a STEM field. Couple this with the fact that only 16% of high school graduates show proficiency in STEM skills and interest in pursuing these careers.⁴

These and many other statistics indicate a need to better connect all students to STEM careers in as many ways as possible. One way is through STEM-infused, project-based learning opportunities designed to inspire students at an early age to learn about and experience real-world STEM careers they might not normally hear about in school.

TERC's *SportsLab* provides a connection between school, STEM careers, and the real world. *SportsLab* is a project-based sport research and design challenge created by EdGE, the Educational Gaming Environments group at TERC. In *SportsLab*, teams uncover the connection between STEM learning and 21st century skills, and explore connected careers in sport research and product design as they create and propose a shoe design for the sport of Parkour.

Par·kour

/pär'kōor/ noun
plural noun: **parcours**

The activity or sport of moving rapidly through an area, typically in an urban environment, negotiating obstacles by running, jumping, and climbing.

Minorities are hugely underrepresented in STEM careers

2.2%
Latinos



2.7%
African Americans



3.3%
Native Americans



Source: Smithsonian Science Education Center, 2018

What is SportsLab?

SportsLab takes place in a collaborative game-like, interactive online environment, supported by hands-on activities in the real world. Using avatars, teams explore a digital world, experience a narrative story, interact with in-game characters, and access various tools and resources through a series of milestones and missions to help them design a new Parkour shoe to enter in a design competition. *SportsLab* teams do each of the following:

- **Understand connections** and consider how modern athletes depend on and can benefit from the connection between STEM concepts and performance.
- **Explore form and function** to create a *Museum of Kicksology* to uncover how the form and function of a shoe is influenced by the sport or activity for which it is designed.
- **Apply the design thinking process** to create a shoe that meets the needs of an athlete and integrates research findings.
- **Collect and make sense of data** for a variety of shoe outsoles, using a classroom traction tester to provide insights into their team's outsole designs.
- **See how data from a top female Parkour athlete**, collected specifically for *SportsLab* in the Nike Sport Research Lab, can be turned into data visualizations of impact forces for her sport-specific moves.
- **Use data visualization** to inform their midsole design.
- **Explore careers** in sport research, product development, marketing, and other industry-related jobs.

These and other explorations are introduced through the storyline of the online character Eva, a kid who loves Parkour more than any science class. She finds an interest in STEM only when she sees how it relates to her world. In particular, she's interested in how STEM insights can help her hack her own shoes to get better at her sport. After visiting a local makerspace to meet with a shoe designer, she comes up with an idea for a design challenge and "creates" *SportsLab* to get other kids motivated to help crowdsource her quest for a killer Parkour shoe. Eva introduces "players"—teams of two to four students—in her online world to other characters she



Figure 2. In the Museum of Kicksology, SportsLab participants uncover various STEM connections to shoes designed for different sports.

has met that lead them to explore various locations, including a sport research lab, makerspace, parkour gym, and sporting goods store. As teams move through these locations, they uncover embedded assets, including background videos, data-sets, and challenges. Teams apply these assets to real-world activities that culminate with each team creating a set of Final Deliverables—an inspiration board and product pitch for their Parkour shoe design. Each team's *Final Deliverable* is uploaded to the GrindArena in the *SportsLab* environment where the top designs are shared for other teams to see and for judges to evaluate. The top designs can win prizes from *SportsLab*'s industry partner, Nike.



SPORTSLAB Opening the Door to 21st Century Careers

Where is *SportsLab* headed?

In addition to creating and running *SportsLab*, EdGE has been conducting research on how *SportsLab* engagement impacts improvements in participants' STEM, Information and Communications Technology (ICT), and 21st century skills dispositions and career awareness. We're now entering the last phase of our research, working to determine if *SportsLab* will better engage youth, many of whom may not think of themselves as STEM or ICT oriented. So far, our approach shows promise. Results from our first design challenge support *SportsLab*'s appeal to a broad diversity of learners. One educator of at-risk students said of the program, "For me to see how much my students are engaged in learning, makes my heart sing." Our research indicates that *SportsLab* can be implemented in a wide range of formal and informal education settings where, as another teacher said, "Students felt like they were not playing at a career but got to see what that career might be like." Students liked the narrative of Eva's story and the environment, which helped make the activities feel more connected. Students found this approach different from most science activities. Participants especially liked the hands-on, real-world activities tied to sport research and design.

SportsLab's project-based approach not only helps engage kids interested in sports—pro or not—but also kids already in love with STEM or interested in subjects such as art or design. Just as a design team at a sport product company comprises individuals with a diverse set of skills, a *SportsLab* design team benefits from and draws on each individual's strengths as they collaborate to create their final shoe design.

While we are wrapping up our research, we are looking toward the future with an eye on creating a nationwide *SportsLab Design Challenge*. We hope to secure an industry partner with an interest in STEM education and an education



Figure 3. A *SportsLab* participant collects data for various outsole designs using a classroom traction tester modeled after how researchers test for coefficient of friction in the real world.

partner that sees the value of national competitions. Securing such sponsors will let us extend *SportsLab*'s reach to a more diverse community of educators and learners in formal and informal settings. *SportsLab* will move beyond the sport of Parkour to include design challenges for different sports-related products, including adaptive sports products that integrate activities and insights from education and industry. We envision a *SportsLab* that will be able to respond to changes in STEM and technology, reflecting current as well as future careers.

Interested educators can learn more about *SportsLab*'s current work by visiting the EdGE website (<http://edge.terc.edu>). If you want to learn more about *SportsLab*'s research or its future beyond our grant, contact us via our website or e-mail: sportslab@terc.edu.

"For me to see how much my students are engaged in learning, makes my heart sing."

— Educator of at-risk students



Who knows? Maybe the kid in your class who likes to draw may decide to focus his energy on learning how STEM can improve skills and career choices. Maybe he will design the next killer sneaker. Or if that kid who loves sports turns pro, perhaps her understanding of the importance of STEM to performance may inspire her to give back to education, as the real LeBron James did by funding a school and a better future for others. *SportsLab* can build a future that helps kids see the connection between mind and body, learning and sport, and fulfilling careers.


This work was supported by the National Science Foundation under grant 1311901. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

ENDNOTES

1. NPR (2015). How Likely Your Athletic Kid Will Turn Pro. Retrieved from <https://www.npr.org/sections/health-shots/2015/09/04/432795481/how-likely-is-it-really-that-your-athletic-kid-will-turn-pro>
2. NCAA (2018). Probability of Competing Beyond High School. Retrieved from <http://www.ncaa.org/about/resources/research/probability-competing-beyond-high-school>
3. Smithsonian Center for Science Education (2018). The STEM Imperative. Retrieved from <https://ssec.si.edu/stem-imperative>
4. UC Davis STEM Portal (2018). STEM and US Job Market. Retrieved from <https://stem.ucdavis.edu/stem-and-us-job-market/>

THANKS

The SportsLab Team would like to thank the EdGE at TERC team, the Nike Sport Research Lab, Vernier Software and Technology, Exeter Research, New Knowledge Organization, and FunAtomic for their support and work on *SportsLab*. We'd also like to thank all the educators and students that have provided feedback along the way. Lastly, a thank you to Brandee Laird of Parkour Visions, a truly inspirational athlete who gets the balance between art and science.




Eva

Museum of Kicksology

You might be a sneakerhead and know everything there is to know about a brand or style of shoe. Me? I never really thought about shoes until I wanted to design a Parkour shoe. There is a lot to know. In this activity you will make an exhibit for a Shoe Museum to help you think about design and the science behind kicks of all kinds.

Materials you'll need to complete the Mission:

1. As many different shoes as you can get your hands on.
2. Materials to create your "museum exhibit" for one or more shoes—note cards, stands, boxes to set shoes on, anything that will let you mount and share your exhibit of shoes.
3. A digital camera to take a picture of your exhibit as well as others you find interesting. You will upload a picture of your exhibit to SportsLab.




Dennis Maker

What you need to do:

Work with a partner and select a shoe from the shoes provided, or use your favorite shoe to display as part of your team's exhibit in a Shoe Museum. Your exhibit needs to have (at least!) the following components:

- Labels for as many parts of the shoe as you can identify.
- Description of the sport or activity you think the shoe is designed for.
- Highlighted features for specific benefits the shoe might provide. For example, a wool lining (feature) in a snow shoe might keep the foot warm (benefit).
- If the shoe has been worn, point out what the wear pattern tells you about the person who might have worn it. For example, if a running shoe is worn out on one heel but not the other, it tells you something about the running form of its owner.
- Ways you think science might have influenced the design.
- The Team Name and Player Names of the exhibit designers.

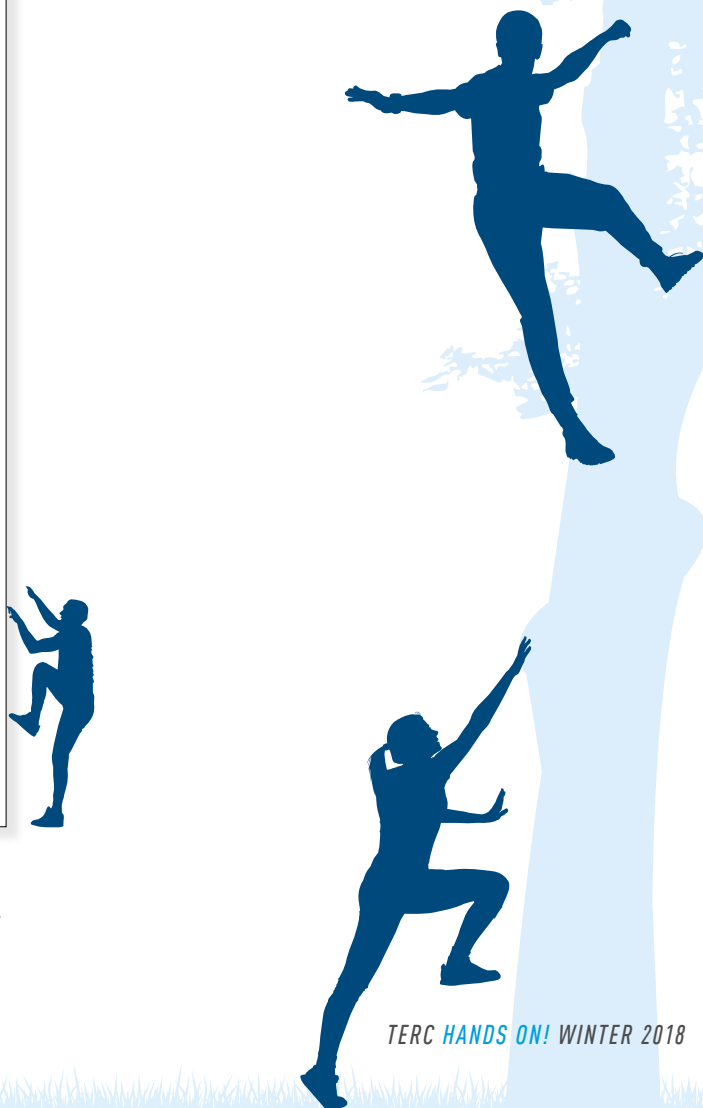
Your exhibit will be part of a Museum of Kicksology to share with others.



Lori
Shoe Store Owner

Lots of people collect kicks, some like to collect all of the shoes for a particular sport and brand—like all Jordan's. Some just like to collect tons of shoes. The more I look at shoes, the more I wonder about which features provide a true benefit and which are just marketing, something that looks good, but doesn't contribute to the performance of the athlete. What do you think?

Figure 4. The Museum of Kicksology activity provides information to student teams on how to set up their own exhibit to help them learn about parts of shoes as well as uncover design considerations specific to a particular sport or activity.





A DURABLE FEAST:

Notes from the

TERC

John Dewey
Group

After 15 Years

To celebrate its 40th year, TERC hosted a symposium entitled “Science Education for a Thriving Democracy.” Our work includes more than science education, of course, but TERC researchers all share a commitment to educational work that supports and encourages teachers and learners to be whole-hearted, creative, and reflective participants in democratic society. Though we draw on the ideas and designs of a variety of thinkers, many of us at TERC find the work of John Dewey to be a rich source of ideas, challenge, and inspiration.



John Dewey (1865-1952) thought and wrote in an era of wars, mass immigration, social unrest, and globalization. He had a deep commitment to democracy as a process of daily life, and to philosophy as a tool for understanding and improving social, as well as individual, growth. An early leader in experimental psychology, Dewey brought his project of radical reconstruction to every branch of philosophy—ethics, logic, aesthetics, social psychology, and more. He put his principles on the line by engaging with organizations working for peace and for social and economic justice. Though people sometimes think about him as primarily a philosopher of education, it might be better to say that he saw education as a key concern of philosophy; it shows up in most of

his works, one way or the other, and occasionally takes center stage, as in the classic *Democracy and Education* (1916).

Since 2003, a group of TERC researchers has been meeting roughly once a month to read Dewey. We often share chocolate (and an occasional celebratory glass of wine), check in on each other's lives, argue, joke, and think together. This long-running symposium, or philosophical feast, has changed its participants—and perhaps through us, changed TERC a little bit as well. Fifteen years on, four core members of the group share about the Dewey group, and how this long-running conversation has affected their work and outlook.

George Hein. TERC Trustee.



Although I took quite a few philosophy courses as an undergraduate in the early 1950's, Dewey was never mentioned. (He was still alive but out of favor then.) I became interested in science education and joined the Education Development Center as an academic chemist in the

late 1960's. There at the Elementary Science Study project, the spirit of Dewey's progressive educational views was palpable, but I didn't recognize it at the time. It was only when I began a serious career as an educator interested in informal science activities at museums that I began to appreciate how relevant Dewey was to our work.

In November 2003, I gave a lunch talk at TERC on "Dewey and Museums." Somehow, that talk led to a conversation with Brian Drayton and the beginning of the Dewey reading group. In the ensuing 15 years the

"Dewey's overall approach to philosophy—that experiences are our primary and only source of knowledge..."

group has read all of Dewey's major works (some more than once), as well as countless essays, shorter pieces, and some books about Dewey's work.

Dewey's overall approach to philosophy—that experiences are our primary and only source of knowledge; that aspects of our existence (such as mind and body or thought and action) are not separate entities but associated; and that making meaning of our lives is a never-ending process that requires both constant inquiry and reflection—has contributed significantly to my work. It informs both the practical aspects of how I understand data gathered from qualitative evaluation work and my continuing efforts to associate education in a democracy with social justice issues.

Dewey's ideas about experience, existence, and inquiry are also expressed frequently in TERC's projects; for example, in Kids Network, in which students collected real data and interacted with scientists; in Chèche Konnen, in which middle school children chose and carried out a research project of interest to them; or in Ricardo Nemirovsky's museum exhibit that allowed children to embody mathematical formulae through physical actions.



Brian Prayton, Co-director, Center for School Reform at TERC



When I first came to TERC, I was too caught up in the details of my project work to feel the need for a “philosophy.” Luckily, I was surrounded by inquiring minds

who introduced me to a kaleidoscope of names—Piaget and Duckworth, Bruner and Papert, Vygotsky, Hawkins, and ... Dewey? I’d heard about Dewey from a childhood mentor, but all I had were snippets about “experience” and “learning-by-doing.” I got little further until the late ‘90s when Joni Falk and I were researching how middle-school science teachers understood and enacted inquiry.

Wondering what “inquiry” meant anyway, I came across a quote by Dewey about a persistent issue in science education (in “Science as Subject-matter and as Method”):

“One of the most serious difficulties that confronts the educator who wants... to do something worthwhile with the sciences is their number and the

indefinite bulk of the material in each... There is at once so much of science and so many sciences that educators oscillate, helpless, between arbitrary selection and teaching a little of everything.”

Written in 1910, this statement seemed so penetrating, so full of implication, and so pertinent to issues arising from the standards movement that I had to find out more. The more I read Dewey on education, and on inquiry, the more I realized that I was on the shores of a broad continent of ideas, mapped out by a radically curious and purposeful explorer. It was about that time that I had the encounter with George Hein which led to the founding of TERC’s Dewey reading group.

The years of Dewey immersion have been a remarkable gift, or rather a gift basket. Dewey always invites the next question, always asks about consequences, always dares to think aloud. His radicalism is ever-surprising, and his combination of idol-smashing and meaning-making is relentless. The Dewey group has given me an experience of inquiry in company, mutual education through dialogue—as I think he would have liked. My thinking, my writing, and my own inquiries have come to be flavored and retuned by this long engagement with Dewey and with some friends.

Gilly Puttick, Senior Scientist



The writings of John Dewey have been a steady presence in my work at TERC. Traditional science education has often characterized scientific inquiry for students as a list of steps in “the scientific method.”

Yet, even to those of us at TERC who advocate an inquiry practice that is more closely aligned with how science is actually done, Dewey would offer a more

profound way to capture the essence of inquiry. He would say, “Let students use ideas as hypotheses, observe the consequences they produce when acted upon, and reflect on the ideas, activities and consequences to extract meaning ... and to serve as the ‘capital stock for intelligent dealing with further experiences’” (*Logic: The Theory of Inquiry*).

Thanks to the Dewey reading group, this Deweyan perspective has increasingly become a way of life for me. Using ideas as hypotheses can ensure that I approach events with an open mind, and I try to be a careful observer. This practice is often aspirational, of course; for example, sometimes I find myself observing without really seeing. Regularly digging deeper into Dewey with colleagues serves its purpose



by causing me to pause, reflect, and make sense of the world again.

Perhaps most important is Dewey's emphasis on the relevance of education to building and maintaining a democratic society and to the improvement of social conditions. How opposite Deweyan thinking is to our current national climate! For example, Dewey wrote,

"The undisciplined mind is ... prone to assertion. It likes things undisturbed, settled, and treats them as such without due warrant" (*Democracy and Education*). Notes I wrote in the margin of the relevant chapter that day listed, "conservatism, patriarchy, high-stakes testing, religion." Our conversations have always been as wide ranging as this.

Paul Wagoner Lab and Demonstration Coordinator



I suspect I am a dilettante of philosophy, and perhaps of religion and mathematics as well. All three fields offer the promise of surprising and fundamental realizations that underlie and govern the possibilities of our lives together.

Each has been an object of intense scrutiny and has left records for us to ponder.

I want to know something of these fields, because I believe that deep principles can be found to guide our thinking and our actions. Because those fields are so fundamental, each of us chooses (knowingly or not) a point of view and a depth of exploration. In the case of mathematics the choice is mostly about how far to go; there is good agreement regarding fundamentals,

relatively few areas are in dispute, and even esoteric fields are rigorously connected back to fundamentals.

However, in the cases of philosophy and religion I find scores of points of view, often obscure, contradictory, and elaborated to baroque sophistication. The pageantry can be enjoyable, but why would I choose to align myself with one rather than another? I look for points of view that are at least mostly in agreement with the deepest values I have been able to discover so far and that offer insights that feel important. (So much the better if those insights are challenging and demand action and deeper study.)

Dewey's philosophy and his writings are grounded in the real and practical. He outlines his view of the role and value of philosophy and traces the development of Western philosophy through the ages, shining a penetrating spotlight on outmoded notions. As he follows through the implications of his views, he leads us to see the crucial role of the ways we live together, which are both the sources of our understanding and our opportunities for productive action: if we were isolated we would hardly know or understand anything, and it is basically only together that we can do anything good.

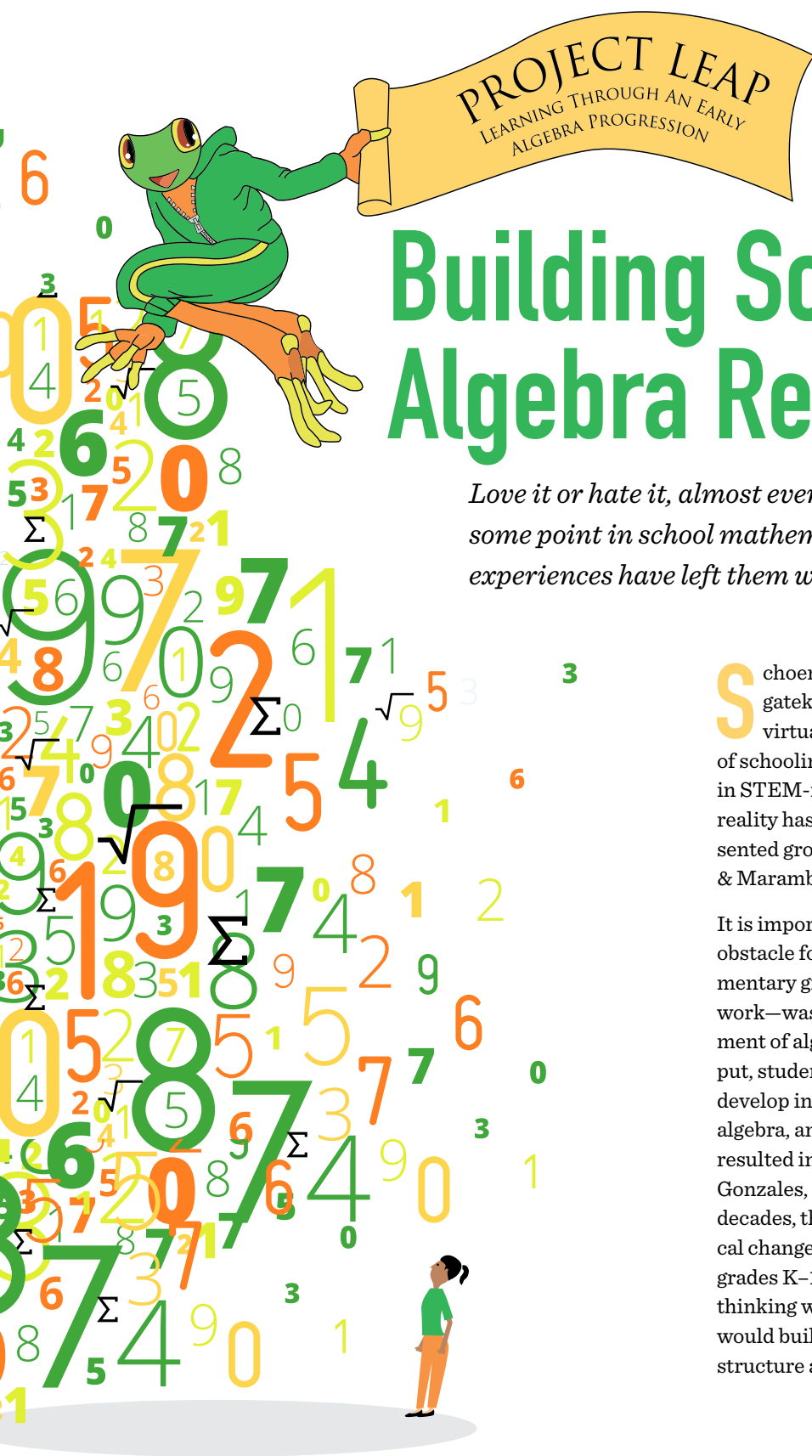
This fall, the Dewey reading group is starting its second reading of *Experience and Nature*.

If you are at TERC or can get here on a lunch break from time to time, you are welcome to join us. Or maybe you'll find a different feast to join or host. A great virtue of a reading circle like ours is that it is not project-related, though it may nourish some project work; nor is it "professional development" in the usual sense. Such groups are free, social, unpredictable, and undertaken for their own sake, for the mere delight of the thing, and for serious, growthful play.

Most of all, they feed the imagination, and John Dewey would be glad to hear it—and glad to join in.

REFERENCES

- Dewey, John. (1910). Science as subject-matter and as method. *Science*, 31, 121-217.
- Dewey, J. (1916). *Democracy and education: An introduction to the philosophy of education*. New York: Macmillan.
- Dewey, John (1925). *Experience and nature*. McCutchen Press.
- Dewey, John. (1938) *Experience and education*. New York, Macmillan.
- Dewey, J. (1938). *Logic: the theory of inquiry*. Oxford, England: Holt.



Building Solutions for Algebra Readiness

Love it or hate it, almost everyone has had a brush with algebra at some point in school mathematics. For too many students, these experiences have left them with a sense of failure and even dread.

Schoenfeld wisely observed that algebra has become a gatekeeper, “an academic passport for passage into virtually every avenue of the job market and every street of schooling,” that has pushed students out of opportunities in STEM-related careers (Schoenfeld, 1995, p. 11). This reality has particularly impacted students in underrepresented groups (Moses & Cobb, 2001; Museus, Palmer, Davis, & Maramba, 2011).

It is important to understand how algebra developed into an obstacle for so many students. Historically, the focus in elementary grades on arithmetic—particularly, computational work—was followed by an abrupt and largely shallow treatment of algebra in secondary grades (Kaput, 2008). Simply put, students did not have the time or conceptual space to develop insights into the deeper, more abstract elements of algebra, and their (at best) superficial knowledge of algebra resulted in widespread failure in school mathematics (Stigler, Gonzales, Kawanaka, Knoll, & Serrano, 1999). In recent decades, the breakdown of this approach led to calls for radical change in teaching and learning algebra in the US to a grades K–12 approach, in which the development of algebraic thinking would begin in elementary grades in ways that would build naturally on children’s informal intuitions about structure and relationships.

The prospect of this shift raised significant questions. What would such an approach look like in the elementary grades? Would it amount to “pre-algebra” ideas repackaged for young children? Would young children even be capable of thinking in ways that have traditionally been viewed as possible only for older students? What impact, if any, would such an approach have on students’ algebra readiness for secondary grades? And, given that elementary teachers would be at the forefront of reform in algebra education, how should they be prepared to build authentic algebra learning environments that would not reinforce past student failures? A number of researchers have worked to address these questions over the past several decades,¹ and while open questions remain, we currently have a much better picture of the potential for *early algebra*² to alleviate algebra’s gatekeeper status.

What Is (Early) Algebra?

Likely, one of the most important questions in reconceptualizing algebra for elementary grades is “What is the ‘algebra’ we want young children to learn?” Early algebra is not algebra early (Carragher, Schliemann, & Schwartz, 2008). In particular, the focus of early algebra is not on manipulating algebraic equations and expressions, too often a major component of typical algebra classes in secondary school mathematics. Project LEAP³ takes a view of early algebra as a set of core thinking practices (Kaput, 2008): generalizing, representing, justifying, and reasoning with mathematical structure and relationships (Blanton, Brizuela et al., 2018). Noticing mathematical structure and relationships—such as the Commutative Property of Addition or a function that depicts how two quantities vary in relation to each other—and representing these generalizations through words, algebraic notation, tables, graphs, or pictures, is often viewed as the heart of algebraic thinking (Cooper & Warren, 2011; Kaput, Blanton, & Moreno, 2008). However, the two practices of justifying (or refuting) generalized claims and reasoning with generalizations to build new mathematical knowledge are equally important components of algebraic thinking. For example, students might build arguments as to why the sum of two odd numbers is even and then use the claim that “the sum of two odd numbers is even” to reason in new ways about the sum of three odd numbers. They might use properties of operations they notice, such as commutativity or associativity, to reason in more strategic ways in computational work. All these practices provide an important framework that lays the foundation for algebra.



CORE ALGEBRAIC THINKING PRACTICES

- Generalizing
- Representing
- Justifying
- Reasoning with
mathematical structure and relationships

The LEAP Curriculum

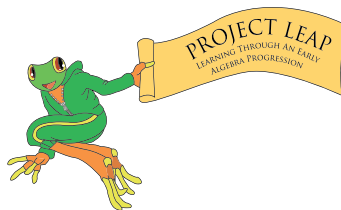
Over the last decade, our team⁴ has worked to integrate these core algebraic thinking practices into the design of an early algebra curriculum for grades K–5 and to examine its impact on children’s algebra understanding and their algebra readiness for middle grades. Currently, we have completed the LEAP curriculum for grades 3–5, a sequence of 18 lessons for each grade level, along with grade-level assessments to measure students’ learning as they progress through the sequence.

The curriculum engages students in the four core algebraic thinking practices across different important areas of mathematical content, using increasingly sophisticated ideas, concepts, and representations. The design of the curriculum uses a learning progressions approach (Clements & Sarama, 2004) that incorporates empirical, classroom-based research on levels of growth in children’s thinking about algebraic thinking practices and concepts. Learning progressions, which are increasingly endorsed for their potential to inform the design of coherent standards, curricula, assessment, and instruction (CCSSI, 2010; Daro, Mosher, & Corcoran, 2011), provide an important research paradigm for our work.



Figure 1. Students write about their mathematical ideas in LEAP classes.

Building Solutions for Algebra Readiness



Lessons begin with a brief “Jumpstart” to review previous concepts or prompt students’ thinking about new concepts, then transition into small-group investigations in which students explore concepts that engage them in algebraic thinking practices. They conclude with a whole-group discussion of students’ findings and a “Review and Discuss” that serves as a formative assessment. Across grades 3–5, the curriculum develops increasingly sophisticated understandings of algebraic concepts and practices, emphasizing the development of meaning for algebraic ideas by engaging students in explaining and justifying their thinking, both orally and in writing.

Can Children Think Algebraically?

Through experimental studies, we have found that students who are taught the LEAP curriculum as part of their regular math instruction significantly outperform students who receive only regular, arithmetic-focused instruction on growth in understanding of core algebraic concepts and practices (Blanton et al., 2017; Blanton et al., 2018). LEAP students are significantly more able to interpret the equal sign as a relational symbol, recognize properties of operations and represent them with variable notation, build arguments for mathematical claims that are increasingly general and more sophisticated than arguments that test numerical examples, recognize unknown quantities in mathematical situations and represent them as algebraic expressions, and generalize functional relationships and represent them with words and variable notation. Surprisingly, we have also found that students are more successful in representing function rules with variable notation than with their own words, underscoring the argument that variable notation can be an important tool in even young children’s algebraic reasoning.

Consider the following example of growth in students’ ability to generalize and represent relationships—two core algebraic thinking practices. In the “Brady Problem” (see Figure 2), students were asked a variety of questions, including whether they could find a relationship between the number of desks and the number of students that could be seated at the desks and to represent this relationship with words and variable notation. Table 1, which records one student’s responses across grades 3–5, shows increasingly sophisticated ideas that grow from not recognizing a pattern or relationship at the beginning of third grade, to a clear understanding of a correspondence functional relationship in fifth grade.

Our most compelling finding, however, is that the effectiveness of the LEAP curriculum also holds for those students who come from disadvantaged and demographically diverse backgrounds (Blanton et al., 2018): LEAP students from low SES (socioeconomic status) and demographically diverse schools significantly outperform their peers in similar schools where only the regular arithmetic curriculum is taught. For us, this shows great promise in LEAP’s potential to ameliorate algebra’s gatekeeper effect and broaden the STEM pipeline to include underrepresented groups.

We are optimistic that curricula such as LEAP can continue to change the way students learn algebra and lead to more opportunities for success in mathematics for *all* students. Currently, we are working on two core goals: first, expand LEAP into grades K–2; and second, identify and test design principles that address the needs of students with learning difficulties and differences and that encourage culturally responsive teaching. The design of a comprehensive, grades K–5 approach to teaching and learning algebra that addresses the needs of all learners will, in our view, make an important contribution to developing children’s algebra readiness.

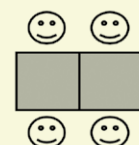
Figure 2. The Brady Problem.

**Brady is celebrating his birthday at school.
He wants to make sure he has a seat for
everyone. The desks are square-shaped.**

He can seat 2 people
at one desk in the
following way:



If he joins another desk
to the first one, he can
seat 4 people:



If he joins another desk
to the second one, he
can seat 6 people:

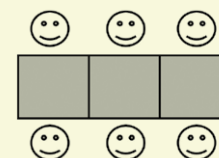


Table 1. One student's response to the Brady Problem across grades 3–5.

Timepoint	Relationship (Words)	Relationship (Variables)
Beginning of Grade 3	I Put 3 desks and four kids on each desk and there is 12 kid	?
End of Grade 3	the # of desks count by 1 # of people count by 2.	?
End of Grade 4	It is always plus its self	$A \times 2$
End of Grade 5	The number of people is double the number of desks.	$d \times 2 = p$

ENDNOTES

- ¹ For an overview of the state of research on children's algebraic thinking, see Cai & Knuth (2011); Carraher & Schliemann (2007); Kaput, Carraher, & Blanton (2008); and Stephens, Ellis, Blanton, & Brizuela (2017).
- ² By early algebra, we mean algebraic thinking in elementary grades.
- ³ The LEAP (Learning through an Early Algebra Progression) moniker is based on the use of a learning progressions approach in the curriculum's design.
- ⁴ The Project LEAP team is a collaboration among researchers at TERC, the University of Wisconsin Madison, the University of Texas Austin, City College of New York, and Merrimack College.

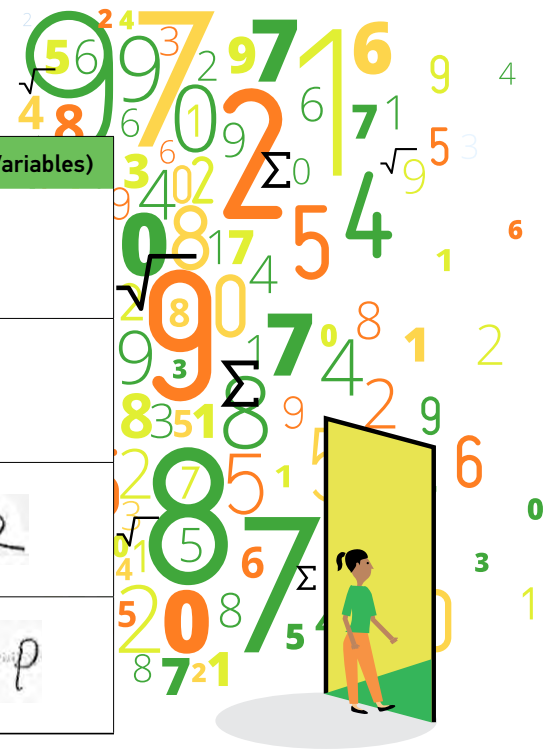
ACKNOWLEDGEMENT

The research reported here was supported in part by the National Science Foundation under DRK-12 awards 1219605 and 1219606 and by the Institute of Education Sciences, U.S. Department of Education, through grant R305A140092. Any opinions, findings, and conclusions or recommendations expressed are those of the authors and do not represent the views of the National Science Foundation or of the Institute of Education Sciences or the U.S. Department of Education.

REFERENCES

- Blanton, M., Isler, I., Gardiner, A., Stephens, A., Knuth, E., & Stroud, R. & Strachota, S. (2017). *Growth in children's algebraic thinking: A grades 3–5 early algebra intervention*. Paper presented at the National Council of Teachers of Mathematics Research Conference, San Antonio, TX.
- Blanton, M., Brizuela, B., Stephens, A., Knuth, E., Isler, I., Gardiner, A., Stroud, R., Fonger, N., & Stylianou, D. (2018). Implementing a framework for early algebra. In C. Kieran (Ed.), *Teaching and learning algebraic thinking with 5- to 12-year-olds: The global evolution of an emerging field of research and practice*. (pp. 27–49). Hamburg, Germany: Springer International Publishing.
- Blanton, M., Stroud, R., Stephens, A., Gardiner, A., Knuth, E., Stylianou, D. (2018). *An Early Algebra Intervention for Upper Elementary Grades*. Paper presented at the National Council of Teachers of Mathematics Research Conference as part of the Symposium Rigorous Research to Improve Learning in Elementary Math for All Students, Washington, DC.
- Cai, J., & Knuth, E. (Eds.). (2011). *Early algebraization: A global dialogue from multiple perspectives*. Heidelberg, Germany: Springer.

- Carraher, D. W., & Schliemann, A. D. (2007). Early algebra and algebraic reasoning. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (Vol. 2, pp. 669–705). Charlotte, NC: Information Age Publishing.
- Carraher, D. W., Schliemann, A. D., & Schwartz, J. L. (2008). Early algebra is not the same as algebra early. In J. Kaput, D. Carraher & M. Blanton (Eds.), *Algebra in the early grades* (pp. 235–272). Mahwah, NJ: Lawrence Erlbaum Associates.
- Clements, D. H. & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, 6(2), 81–89.
- Cooper, T., & Warren, E. (2011). Years 2 to 6 students' ability to generalize: Models, representations, and theory for teaching and learning. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 187–214). Heidelberg, Germany: Springer.
- Daro, P., Mosher, F., & Corcoran, T. (2011). *Learning trajectories in mathematics: A foundation for standards, curriculum, assessment, and instruction*. Retrieved from <http://www.cpre.org>.
- Kaput, J. (2008). What is algebra? What is algebraic reasoning? In J. Kaput, D. Carraher, & M. Blanton (Eds.), *Algebra in the early grades*. Mahwah, NJ: Lawrence Erlbaum Associates/Taylor & Francis Group.
- Kaput, J. J., Blanton, M. L., & Moreno, L. (2008). Algebra from a symbolization point of view. In J. J. Kaput, D. W. Carraher, & M. L. Blanton (Eds.), *Algebra in the early grades* (pp. 19–55). New York: Lawrence Erlbaum/Taylor & Francis Group & National Council of Teachers of Mathematics.
- Moses, R. P., & Cobb, C. E. (2001). *Radical equations: Math literacy and civil rights*. Boston: Beacon Press.
- Museus, S., Palmer, R. T., Davis, R. J., & Maramba, D. C. (2011). *Racial and Ethnic Minority Students' Success in STEM Education*. Hoboken, NJ: Jossey-Bass.
- Schoenfeld, A. H., (1995). Is thinking about 'Algebra' a misdirection? In C. Lacampagne, W. Blair & J. Kaput (Eds.), *The Algebra Colloquium. Volume 2: Working Group Papers* (pp. 83–86). Washington, DC: US Department of Education, Office of Educational Research and Improvement.
- Stephens, A. C., Ellis, A. B., Blanton, M., & Brizuela, B. M. (2017). Algebraic thinking in the elementary and middle grades. In J. Cai (Ed.), *Third Handbook of Research on Mathematics Teaching and Learning* (pp. 386–420). Reston, VA: National Council of Teachers of Mathematics.
- Stigler, J.W., Gonzales, P., Kawanaka, T., Knoll, S., & Serrano, A. (1999). *The TIMSS videotape classroom study: Methods and findings from an exploratory research project on eighth-grade mathematics instruction in Germany, Japan, and the United States*. (NCES1999-074). U.S. Department of Education. Washington, DC: National Center for Education Statistics.





DOING THE MATH with Paraeducators

Paraeducators (paras) are an important and underutilized resource with great potential to impact mathematics learning in elementary classrooms.

EVOLUTION OF THE PARAEDUCATOR ROLE

Paraeducators (paras) are an important and underutilized resource with great potential to impact mathematics learning in elementary classrooms. Yet there is a widespread lack of preparation and support for them around subject matter knowledge, instructional strategies, and skills to address students' different learning styles and needs. In one research study (Houssart, 2013), after being asked to teach a lesson on subtraction with no preparation or guidance, one para lamented, "I just taught them by demonstration ... [I told students], 'This is how you do column subtraction,' and I just felt like the worst teacher in the world ever."

Unfortunately, this para's experience is not unique. Paras are paid by the hour for time spent with students only. They do not have regular access to the same curriculum resources as classroom teachers, or many opportunities to compare their challenges, strategies, and approaches. Further, the classroom teachers they work alongside seldom receive guidance about supervising and collaborating with paras (Ashbaker and Morgan 2012; Giangreco et al. 2012).

"Paraeducator" has replaced titles such as teacher aide, teaching assistant, instructional aide, and paraprofessional, in recognition that these professionals work alongside teachers. The National Resource Center for Paraeducators (NRCP) has noted that, at one time, paras' roles "... were limited to performing routine monitoring and clerical tasks." Now, they "work with some of our most challenging students... [and] are being tapped to fill critical teacher shortages" (<http://www.nrcpara.org/paranews>). As the roles and responsibilities of paras have changed over many decades, there is a need for professional development that reflects their changing roles and growing need for math content knowledge, instructional strategies and learning communities.

GOALS AND COMPONENTS OF *DOING THE MATH*

In response to studies revealing that paras do not feel prepared for new instructional roles (Breton, 2010), and that they need opportunities that emphasize instructional strategies tied to mathematical concepts, curricular goals, and collaborative skills (Liston et al., 2009), we launched our NSF-funded *Doing the Math with Paraeducators: A Research and Development Project* (DRL 1621151). This professional development (PD) model is designed to increase paras' confidence, knowledge of elementary math ideas, pedagogical strategies, and understanding of how students learn mathematics. We are working with Boston Public Schools paras in grades K-3.

In our PD model, paras are immersed in math content and pedagogy by doing, reacting, and reflecting on a variety of math problems and student work. Then, they analyze and understand the goals, strategies, and resources of the district mathematics program, by planning, engaging in, and analyzing math activities for students.

Doing the Math PD Model

- Thirty hours of PD: three days in the summer and four half-days during the school year
- A half-day session for collaborating teachers
- Follow-up math class observations and debriefing meetings
- Formal reflections on their learning five times throughout the year
- Planning protocols that paras and teachers complete each month

SUCCESSFUL ELEMENTS OF THE PD MODEL

Our first cohort completed the *Doing the Math* program in June 2018. Their ratings of their experiences and project researchers' observations suggest several elements that can make a PD model responsive to and effective for paras. These include:

- 1 Giving time and attention to creating a safe and encouraging learning environment:** Paras need to learn in a context where they do not fear getting a “wrong” answer and can reveal their confusion about math ideas. Once this was established for our cohort, conversations were energetic and productive. Participants were eager to try out new strategies to solve math problems, as well as to share their knowledge, experiences, frustrations, and questions. Their confidence grew over the year; as one para expressed it (Figure 2):
“To meet other paras that [were] in the same boat as me. ... really helped and encouraged that [work on math].”
“I feel a lot more secure about math than I did before.”
- 2 Providing paras with access to the math curriculum and opportunities to understand the curricular goals and philosophical-learning orientation:** This support was especially important, since many paras do not have regular access to curriculum resources. Paras came away with a better understanding of the purpose of and how to teach unit resources, including math games.
- 3 Providing opportunities to do math problems in ways that support math sense-making and enable them to reflect on how this approach differs from one that is procedural:** For instance, paras began to comprehend that solving fraction problems involved more than memorizing a procedure. Rather, it was about understanding number relationships. Teachers working in the classroom with PD participants noted that this was an area of growth; for example:

One teacher noted how the PD strengthened the instructional skills of a para in her classroom:
“She has grown and gained confidence in the material and in what math lessons are about. Last year she couldn’t have done a group on her own...Now she is confident.”

“When discussing equivalent fractions, some kids were having trouble conceptualizing how two fractions can be different but take up the same amount of space inside of a whole. The para helped students to see the relationship by drawing pictures.”



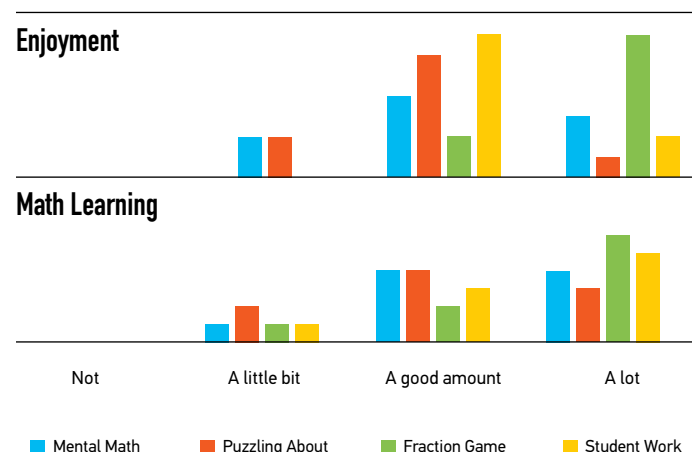
Figure 1. Paras collaborating during PD.

FINDINGS RELATED TO PARAEDUCATOR GROWTH

During the year, we found that paras became more discerning and reflective about their own skills and needs over time. They reported high levels of *enjoyment* as they engaged in math teaching activities. At the same time, they honestly noted the challenges that lay ahead, rating their math learning slightly lower and their *feelings of preparation to teach* lower still. An example of this pattern related to specific PD topics is provided in Figure 2.

Furthermore, our data indicated that the *paras formed a learning community that fostered professionalism and changed paras’ discourse about their students and their work.* Having conversations about their students and working together on math instruction broadened paras’ perspectives about teaching and learning practices across classrooms and grade levels. It helped them think in new ways about working with students who need more support.

Figure 2. Para Perceptions of Growth.



DOING THE MATH with Paraeducators

While the paras in the cohort appreciated that students learn at different rates, many didn't understand what that meant in relation to specific math learning behaviors. Rarely had they been allowed to see specific information about student learning challenges. As a result, some assumed that students fit within two broad categories—those who have special needs and struggle with math and those who do not. Following PD, we discovered that the para cohort had developed *a more nuanced awareness of student needs and appreciated the complexity of learning styles*.

NEXT STEPS FOR *DOING THE MATH*

Working with paras such as Natalia (see below) was a rewarding experience for our staff. She, along with other paras in the cohort, are helping to improve our model as we begin work with a new group. Many of them will serve as mentors to the 2018-19 para cohort and will facilitate analyzing student work sessions. Their feedback on and progress during *Doing the Math* points to the many ways that paras can be supported to build community, foster professionalism, strengthen instructional skills, and change their discourse about their work as

educators. Their growth, enthusiasm, and insight underscores our recognition that paraeducators are a great, but underdeveloped, resource for mathematics learning in elementary classrooms.

REFERENCES

- Ashbaker, B. & Morgan, J. (2012). Team players and team managers: Special educators working with paraeducators to support inclusive classrooms. *Creative Education* 3(3), 322-327.
- Breton, W. (2010). Special education paraprofessionals: Perceptions of preservice preparation, supervision, and ongoing developmental training. *International Journal of Special Education* 25(1), 34-45.
- Giangreco, M. F., Doyle M. B., Suter, J. C. (2012). Constructively responding to requests for paraprofessionals: We keep asking the wrong questions. *Remedial and Special Education* 33(6), 362-373.
- Houssart, J. (2013). "Give me a lesson and I'll deliver it": Teaching assistants' experiences of leading primary mathematics lessons in England. *Cambridge Journal of Education* 43(1), 1-16.
- Liston, A.G, Nevin, A. and Malian, I. (2009). What do paraeducators in inclusive classrooms say about their work? Analysis of national survey data and follow-up Interviews in California. *Teaching Exceptional Children Plus* 5 (5), 2-17. Retrieved from: <http://files.eric.ed.gov/fulltext/EJ967750.pdf>



Introducing Natalia

Natalia, a paraeducator who has been teaching Pre-K – Grade 4 for 22 years provides an example of para growth.

Natalia's ability to explain her own math

problem-solving strategies, listen to her para colleagues, and admit her own confusions encouraged others to do the same. Ultimately, this contributed to building a learning community.

We saw evidence that Natalia applied what she had learned in the PD to her math teaching in order to accommodate the many ways in which students learn. For example, at a visit to her school one of our staff observed Natalia drawing the number line, a tool first introduced to her at a PD session, and using it as an instructional resource to help students solve problems. She told us that she would not have been able to use this approach prior to the PD.

During one session, Natalia shared her strategy for solving $62+38$ efficiently, by changing the value of 62 to 60, and moving the 2 from the 62 over to the 38, making it 40. She explained that children would find it much easier to add $60+40$ visually, than to see it written as $62+38$. Just a few moments later she noted the importance of understanding a colleague's method of mathematical problem solving.

Natalia also asked the students questions that promote mathematical thinking, another emphasis of the PD. These questions included, "Is there an easier way than counting by 1s?" and "Can you use fractions when talking about liquids?" She would then ask further questions, so that the students would have to explain their strategies for solving the math problem. If there was a mistake, Natalia would see it as a learning opportunity, not only for the students who volunteered to share their work, but also for the students in the classroom who were reluctant to explicitly seek help.



Sue Doubler



AN INQUIRY-DRIVEN CAREER AT A GLANCE

Since joining TERC in 1990, Sue Doubler has developed a rich and diverse body of work centered primarily on elementary school science teaching and learning, and on ways that technology can support teachers and students. As her most recent project, "Empowering Teachers Through VideoReView," is winding down, we took the opportunity to talk with Sue about her work and time at TERC.



AFTER YEARS OF EXPERIENCE IN THE WINCHESTER SCHOOL SYSTEM, SUE HAD UNDERTAKEN A PH.D. PROGRAM AT LIVERPOOL UNIVERSITY, IN ENGLAND. AS HER DISSERTATION WAS NEARING COMPLETION, SUE RECEIVED AN EMAIL FROM AN OLD COLLEAGUE THAT SAID, "THERE'S AN AD IN THE 'GLOBE' FOR A POSITION AT TERC. YOU MIGHT BE INTERESTED." SHE WAS, AND THRIVED IN TERC'S CLIMATE:

"A leader at TERC once described TERC's aim as the idea that we should look just over the horizon at what's not yet doable, then not only make it doable, but make it practical," Sue said. "This has always been a guide for me in working at TERC."

This aim permeated her innovative projects, starting with the IBM PSL (Personal Science Laboratory) project. IBM PSL built software, curriculum, and teacher professional development using the PSL system of data-collection sensors. It also brought in a talented team of newcomers who became important to TERC's work thereafter.

"In that early work, when TERC got started with technology, technology was new," Sue reflected. "Few people knew how to use a computer. We were filling a void... Now, technology is part of everything; the technologies we use now are viewed as everyday tools."

Always a teacher at heart, Sue's work drew on her profound understanding of teachers and their context. Her work as part of the PALMS (Partnerships Advancing the Learning of Mathematics and Science & Technology/Engineering) Massachusetts Statewide Systemic Initiative led her to create innovative professional development that supported inquiry as a foundation for learning.

In the years after PALMS, a sequence of projects unfolded that drew on all of Sue's previous experience. In a partnership between TERC and Lesley College, Sue developed the first fully online master's program in K-8 science education. This was followed by TERC's collaboration with the Fulcrum Institute (an early Math-Science Partnership project), the Inquiry Project, and Talk Science. From these, Sue's strategic vision led to the VideoReView project.



"Through our work on other projects we learned that teachers didn't have any way to see their own teaching. That led us to VideoReview, in which we developed new video analysis software, so teachers can capture video of science discussions from their own classroom, study it, and discuss with colleagues," Sue said.

A CENTRAL THREAD THROUGHOUT SUE'S WORK IS INQUIRY IN COLLABORATION:

honoring the questions, and the questioners, and focusing deeply on the phenomena, the bit of the real world that is the occasion for the investigation and the discourse. But she stays rooted in the classroom event, where the "magic" keeps happening, and the human relationships that are the basis of learning.





2067 Massachusetts Avenue • Cambridge, Massachusetts 02140

@TERCtweets terc.edu TERC

Nonprofit Org.
U.S. Postage
PAID
Permit No. 375
Nashua, NH

VideoReView

VideoReView is video-supported professional study in which teachers analyze video of their own science discussions and meet with colleagues to discuss students' ideas and reasoning

Notice ... Analyze ... Respond

- Use video technology to see patterns in students' learning you couldn't see before:
- Notice what your students are thinking
- Analyze where they are on their path toward the learning goal
- Respond more effectively in the moment or upon reflection

HOW DOES IT WORK?

Form a school team of three to four colleagues. Select class science discussions to videotape and study. Share video cases with your team. Use the learning sequence to guide your study.



PLAN a Science discussion



ENACT and video class discussions



STUDY the video using VRV software



MEET with your colleagues in Video Club meetings



THE RESEARCH

We are studying how *VideoReView* helps teachers to notice, analyze, and respond to students' science thinking.



OVERVIEW



METHODS



IMPACT

TO LEARN MORE VISIT: <https://inquiryproject.terc.edu/Videoreview> **OR CONTACT:** videoreview@terc.edu