

QUESTION ROUND



n elementary teacher friend recently challenged me to make the closed question

$$2 + 2 =$$

into a debatable one. Assuming the objective was to assess student understanding of addition, I replied with a few options for the new question:

- What is the best way to model this?
- What is the silliest mistake we can make when solving this problem?
- Who can come up with the coolest story for this problem?
- What is the most interesting way I could add numbers to this problem and still get the same solution?

And the list goes on. By opening up the question to opinions and interpretations, students are not only developing arguments but also increasing their engagement with the problem.

So, how do you start doing this? We have been exploring what debatable moments look and sound like: students engaging with mathematics and each other through clear and brief verbal routines, allowing us teachers to listen closely and minimize our talk time. Now it is time for us to turn our attention to what the teacher needs to do in his or her preparation to make these debatable moments happen. What kinds of questions will engage students in rich mathematical debates? As written, most questions in math textbooks do not spark lively discussion

among our students. So how might we transform, adapt, or create new questions that will engage students in debate about content?

CONTENT OBJECTIVES

First off, I want to emphasize that when a class debates, student learning of math content is at the forefront of the activity. We do not want students to debate just for the sake of saying that our students talk in class. So, we need to write questions with a clear, mathematical objective in mind. For example, suppose you were working with students on domain and range of functions. You would not ask students, *Who drew the messiest graph?* because that may have nothing to do with

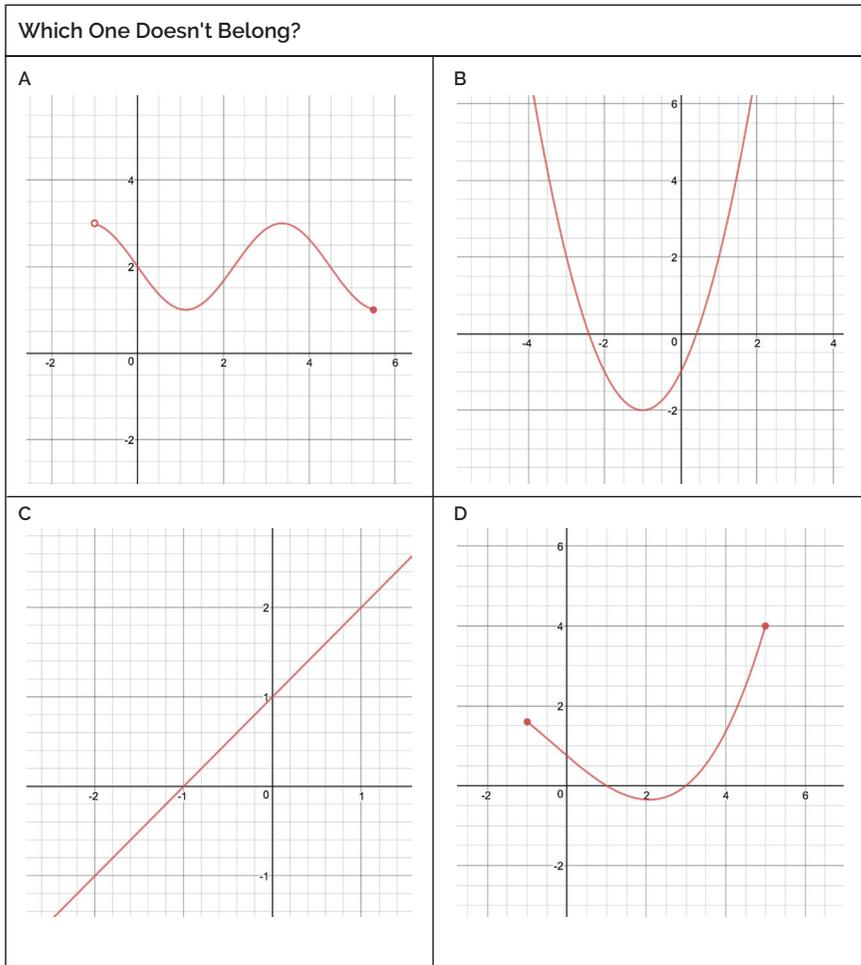


FIGURE 3.1

(POWERED BY DESMOS)

the objective for the lesson. Depending on your goal, you might consider these options instead:

- If your goal is to strengthen student vocabulary around graphs and functions, then you could have students debate a Which One Doesn't Belong? like the one I created in Figure 3.1. It would encourage your students to use words such as *closed*, *open*, *maximum*, and *infinity* (among other great mathy words).
- However, if your goal was to clarify misconceptions about closed and open intervals, you might show examples of incorrect student work and ask, *Who has the best mistake?* as in Figures 3.2A and 3.2B.

Draw a graph with the domain $(-5,5)$ and a range $(-3,3)$.

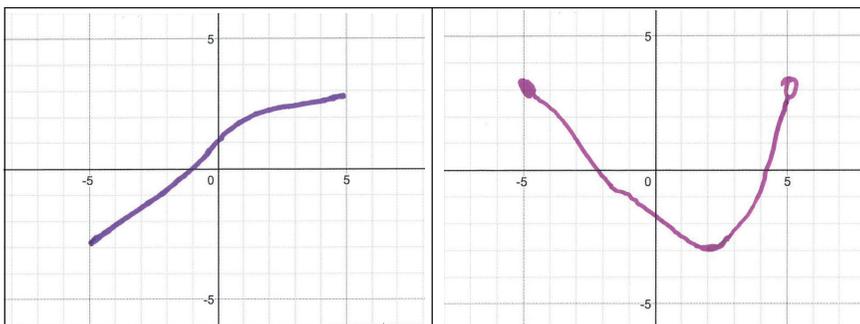


FIGURE 3.2A

FIGURE 3.2B

The objective guides the question. We want the math *learning* to be the point of debating in class. Having clear objectives for each lesson and activity helps keep you laser focused when selecting or writing a debatable moment. Once you have your objective in mind, you can create a debatable question.

WHAT MAKES A QUESTION DEBATABLE?

Many of us struggle to have students discuss math problems because textbook questions traditionally only have one answer. For example:

What is the slope of the line $y = 2x - 8$?

This is a closed question, and it will not engender a debate (let alone much discussion). Closed questions have their place in math class—I am not arguing



to get rid of closed-ended questions. Rather, some of the time, we need debatable questions to get students arguing and critiquing the reasoning of others.

To create debatable moments in class, we need to open questions to students' opinions and give students a chance to have something to say. We need open-ended questions—those with more than one answer—and “opinion-ended” questions—ones that allow for students to develop arguments. Think back to our definition of *claim* in Chapter 2: a controversial statement that has the potential for opposing sides. To realize that potential, we need to ask questions that generate multiple answers or opinions.

Suppose, for example, our objective is to help students understand the importance of slope. Going back to the question about the line $y = 2x - 8$, we could ask questions like: *What is the most important number in the equation of a line?* or *What is the worst mistake you can make when graphing this line?* Take a moment to think of a few examples of arguments that your students might come up with.

What made these questions debatable are the word choices. First, notice how both of the examples allow for multiple “correct” answers. For example, for the first question, students could respond:

Li: I claim that the number 2 is the most important number, and my warrant is that it is the slope. It tells you how much the line slants.

or

Manny: I claim that the -8 is the most important number. My warrant is -8 is the y -intercept and that tells you where to start graphing.

The wording of the question has an ambiguity to it that allows students to take a variety of stances. Additionally, the phrasing has a personal draw that heightens engagement and almost forces opinions to be formed. How can we ask *What is most important?* and *not* have human beings form opinions in their minds?

If students share multiple, solid arguments, your objective is met. Students (and not the teacher) have explained the importance of slope. Even more exciting, the debate usually surpasses your objective because the class has discussed a lot more than just the slope. Students may also have recalled vocabulary like y -intercept and possibly discussed ideas for graphing, such as scaling the axes.

BEST/WORST STARTER

One of the quickest ways to make a question debatable is to start your statement with “The best . . .” or “The worst . . .” For instance, suppose you’re having students practice solving linear equations with distribution such as:

$$\frac{1}{3}(x - 8) = 2$$

If you want students to discuss and debate this problem (either before or after solving it), you could create a debatable moment by giving students one of the following statements and asking them to fill in the blank.

The best first step is _____.

The worst first step is _____.

Some students may prefer to distribute first. Others may talk about dividing by one-third (or multiplying by 3) on both sides. Both are valid first steps, but adding a word like *best* allows students to have an opinion about their method.

Dereka: My claim is that multiplying by 3 on both sides is the best first step, and my warrant is because I hate fractions—I want to get rid of the fraction as soon as possible. Then it is easier for me to do the rest of the math quickly.

As another example, consider my precalculus class. We began by exploring the parameters of sine and cosine graphs for two or three days. I wanted to start the next class with a quick and informal check for understanding, to see how well students were progressing. Initially, I planned to ask students to complete the following:

GRAPH THE FUNCTION

$$y = 8 \sin\left(\frac{\pi}{15}x\right) + 10$$

It was a straightforward problem, meant to assess students' understandings of how each of the numbers in the equation affects the shape of a sine graph. I wanted students to be able to explain the amplitude, period, and vertical shift and how they would scale the axes. However, there is not much room for opinion in this problem, so I employed the Best/Worst Starter idea to change it to a question this way:

THE BEST WAY TO START GRAPHING A SINE FUNCTION IS . . .

$$y = 8 \sin \left(\frac{\pi}{15} x \right) + 10$$

My students' responses to this warm-up were more interesting and revealing than if students had just graphed the function.

Bryana: *My claim is the best way to graph a sine function is to first look at the amplitude, and my warrant is because it gets you the height.*

Xialin: *My claim is the pi 15 thing is the best way to graph a sine function. My warrant is that it's what you use to make the x-axis work.*

Julion: *I claim the plus 10 is the best way to start graphing because that is the starting point. The midline? From there you can just graph a sine graph and use the amplitude.*

Mr. Luzniak: *I love these arguments! Can I pause for a moment and ask someone to clarify one of the recent arguments with our math vocabulary?*

Jennah: *I claim like Julion that the vertical shift is the best thing to graph a sine function, and my warrant is that it tells you how far up to start on the y. Then if you add and subtract 8, the amplitude, you will know how high to go. And low.*

Notice how students were struggling with both the vocabulary they need and with ordering the steps for starting to graph. As the teacher in the room, I interrupted the debate to push for more precise math vocabulary, but I did not

comment on the ordering of steps. Once we had elicited some student ideas in the debate, I asked the class to try to graph the sine function in parentheses. Giving students a few minutes for quick debate not only increased student engagement and argumentation in the class but also helped many students review the main ideas and clear up some misconceptions. Students who were unsure where to start got support from classmates on how to approach graphing sine functions, which increased their access to the mathematics. I was able to listen in on student thinking and get a sense of where my class was. It was a win-win moment for us all.

MORE DEBATE-Y WORDS

Adding a brief Best/Worst statement makes a straightforward question debatable because the students can now have opinions about what is *best* (or *worst*) for the given situation. *Best* and *worst* are not the only words that can invoke opinion, but they are my go-to starters. Following is a fuller list of words that can turn a closed mathematical question or statement into a debatable one.

- | | |
|--------------------|---------------------|
| • Best/Worst | • Weirdest/Coollest |
| • Should/Could | • Hardest/Easiest |
| • Biggest/Smallest | • Most/Least |

These words are superlatives and assertions that push students to form opinions, and these opinions, in turn, create debatable moments in class. Here are some examples of questions teachers have written using these starters:

- What do you think is the *most* common mistake when solving this problem?
- What is the *weirdest* mistake someone could have made on the test?
- What *should* be the last step when graphing a line?
- Which shape do you think will have the *most* area? The *least* perimeter?
- What is the *coolest* parent graph?
- What was the *hardest* topic in this unit?
- What is the *biggest* mistake students could make when adding fractions?

- What is the *easiest* way to estimate the area under this curve?

This list can help you write a debate-worthy question for your next lesson. Keeping your mathematical objective in mind, try choosing a superlative to create a question where students can form opinions. Then, after you try it out in class, reflect on what you learned from your students' responses.

MAKE IT DEBATABLE!

We can write our own original, debatable questions with the previous list. We can also quickly transform a closed question from a textbook, resource, or already developed lesson plan into one that leads to a debate. Some examples of this transformation:

$$\frac{1}{2} + \frac{3}{4} =$$

What is the *coolest* way to visualize this problem?

Solve for x :
 $2x + 3 = 3x - 4.$

What is the *most* common mistake you might see in this problem?

Find the radius of a circle with an area of 24π square units.

What is the *hardest* part about this question?

Solve for x : $0 = x^2 + 3x$

What is the *best* method to solve this equation?

Determine $\sin\left(-\frac{9\pi}{4}\right)$

What *should* be the first step in solving this problem?

Find $f'(4)$ when $f(x) = 3\sqrt{x} + 1$

What is the *biggest* mistake you could make in solving this problem?

With practice, it can become easier, almost second nature, to use our list of superlatives to reword closed questions into debatable ones. Daily! I certainly use them over and over in my lessons.

As useful as these words are, there are other ways we can create debatable questions and moments in the classroom. Let's explore some techniques together.

ALWAYS, SOMETIMES, NEVER

Some of the curricula you use or the resources you rely on already have “thinking” or “discussion” questions written for you. These can often be great questions to use once you have a Talking Routine (like claim and warrant) fixed in your class. I encourage you to use quality premade questions when you can.

One way to ensure you will generate debate about the “discussion” question is to ask students if the statement is always true, sometimes true, or never true. For example, are the following statements always, sometimes, or never true?

- When you add or subtract 10, the ones digit does not change.
- The fraction with the larger numerator is the larger fraction.
- Every shape has a line of symmetry.
- The variable must always be on the left side of the equal sign.
- If Jan gets a 30% raise and Matt gets a 25% raise, Jan got the bigger raise.
- A triangle can have two obtuse angles.
- 4π and $2x\pi$ are co-terminal angles on the unit circle.
- Lisa’s velocity is given by the function $y = 4x$. Zane’s velocity is given by the function $y = 6x$. Therefore, after 5 seconds, Zane will be the farthest away.

These questions often evoke great conversations as students claim that the statement is either always, sometimes, or never true and then try to justify their claims with warrants. For example, you might hear in a geometry class:

Lorn: My claim is that “A triangle can have two obtuse angles” is never true, and my warrant is that that would mean two angles are more than 90 degrees. There is no way that can happen if the angles have to add up to 180.

And in a precalculus class, you could hear:

Melani: I claim 4π and $2x\pi$ are always co-terminal angles, and my warrant is that when you plug in numbers for x you get 2π , 4π , 6π , and so on. These are all co-terminal.

Kyra: My claim is that 4π and $2x\pi$ are sometimes co-terminal angles because when you plug in a fraction like $\frac{1}{2}$ for x , it is in a different place in the unit circle.

What constitutes an acceptable warrant really depends on the experiences of your students and your goals in that moment. Whether or not students have seen formal proofs before, they intuitively know what it means to be convincing. As their teacher, with knowledge of your students, you can push students to increase the rigor. You can set the bar for solid reasoning. Is a counterexample a reasonable expectation? Is a formal proof what you are building toward? With all debates, what students say may be quite rigorous, or you can challenge students to heighten their arguments with more mathematically solid warrants.

USING STUDENT MISTAKES

One of my favorite ways to get students talking and debating in class is to share student mistakes. You can take mistakes from two different students and debate about the *best mistake* or *coolest mistake*. These questions are usually sources of rich discussion and learning for students. Not only that, but they also help create a classroom culture where mistake making is acceptable and a valuable part of the learning process.

For example, if you were to ask students to convert an angle of 5 radians into degree measurement, that would be a closed question. If you wanted to spark a debate, you might include examples of student mistakes, as in Figures 3.3A–3.3C and ask, *Who has the best mistake?*

$$5\pi\left(\frac{180}{\pi}\right) = 900^\circ$$

FIGURE 3.3A

$$5^\circ \times \frac{\pi}{180} = \frac{5\pi}{180} = 0.09$$

FIGURE 3.3B

$$5 \text{ rad} \approx \frac{5\pi}{3}$$

$$\frac{5\pi}{3} \cdot \frac{180}{\pi} = \frac{300}{1} = 300^\circ$$

FIGURE 3.3C

As another example, in my calculus class recently, we were wrapping up a unit on limits. On the day of the quiz, I created a warm-up for the class to debate (see following slide) that consisted of common mistakes students had made on recent homework problems. This debate lasted just over ten minutes, and it was a great review for students who were still struggling with misconceptions. To engage in the debate, students had to first work out the correct answers. (There were a few students who were convinced that one of the problems was actually correct and that I was tricking them.) Then, they had to figure out what could have led to the mistake pictured. Last, they had to develop an argument for which one was the best mistake. This process highlighted a lot of misconceptions, and students were discussing these common errors with one another, clarifying understanding together.

LET'S DEBATE! WHICH ONE HAS THE BEST MISTAKE?

a) $\lim_{x \rightarrow 2} \frac{x^2 - x}{x + 3} = \infty$

c) $\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x - 2} = DNE$

b) $\lim_{x \rightarrow \infty} \frac{4x^2 + x}{9x - x^2} = \infty$

d) $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = DNE$

Note: DNE means "does not exist."

USING STUDENTS' OWN WORDS

I try to have debatable moments, even very brief ones, daily in my class. They can take the form of a short warm-up question or a writing question on a worksheet or quiz. Anytime students have to convince someone—whether that is a classmate, the teacher, or a generic third person—is a debatable moment.

An added bonus of having students regularly debate ideas in math class is that unplanned debatable moments can also come up. Sometimes a student may say something we were not expecting that causes me or the class to pause, think, and begin a spontaneous debate. For instance, I once had a middle school student say:

I notice that when you tear a piece off of a shape, you reduce its area and perimeter.

Is this true? Where did this idea come from? This was a great moment for students to turn and talk for a minute or two. Then, we started a brief Soapbox Debate, where students shared their ideas:

Daquan: *I claim that this is true, and my warrant is that if you take a square and cut off a piece, you get a smaller area.*

Kati: *I claim that this is sometimes true. My warrant is because if you zigzag when you tear off a piece, you get more perimeter.*

This was a brief debate in my class that I cut short and asked students to ponder at home. However, I also could have allowed more time in the moment for students to discuss and explore the statement. Depending on the time you have available and the importance of the topic, you might adjust your lesson to allow this time. Perhaps your students have more misconceptions than you realized, and they need to debate a bit longer, as they work through the confusion. Or, as happened in my case, both the teacher and the students needed time to process the misunderstandings. I tabled the debate until the next class, giving myself an opportunity to follow up with a planned discussion the following day. I used the delay to give myself time to check in with colleagues, gathering information from those who had taught my current students, and collaborating on ways to proceed with my next lesson. It was clear there were many different directions the conversation could take.

When debating the statement, my students got into a discussion of the word *and*, wondering whether they had to explain both perimeter and area simultaneously or if it was two different questions. When I have shared this statement with adults at conferences, we have discussed whether *shape* implies two-dimensional objects only, or if we can explore what happens with three-dimensional objects (or even four-dimensional ones!). We have even gotten into semantic discussions about the word *tear*. What does that mean exactly? Can you tear in three or four dimensions? Is there a precise mathematical term we can use? Look at all that we've had to say about one student's statement. Imagine how validated that student, and hopefully the class, felt after seeing how I value students' conjectures by spending time on them.

ARE CLOSED QUESTIONS EVER DEBATABLE?

Debatable questions have multiple answers and opinions, like *Which one doesn't belong?* or *What is the best method for finding the area of this irregular shape?* These questions have multiple valid answers or approaches, and students can form differing opinions about them.

On the other hand, closed questions have a single answer. Closed questions are not exactly debatable. However, throughout this book, you can see that some of my examples are actually closed questions. I want to clarify that I include them in my list of debate-worthy examples anyway because I know from experience that they can spark good discussion in the classroom. Many students may initially have differing ideas of the solution, and their misconceptions are what cause a debate. For example, consider asking students to discuss the truth of the following statement:

Squaring a number makes the number bigger.

Some students may wonder if they can find a counterexample. However, most students—or even the entire class—may initially feel that the statement is true. Thus, even though the statement has a definite truth value (of false), the class can explore, discuss, and debate. Having an established culture for debate in the classroom makes discussion of these common misconceptions work well.

Other examples of debate-worthy misconceptions include:

- Can two numbers add up to nothing?
- True or false: Adding fractions always gives you another fraction.
- Can the height of a triangle be a slanted line?
- True or false: Square roots always make a number smaller.
- Is there a difference between $-x^2$ and $(-x)^2$?
- Are these all equivalent: $\sin^2 x$, $\sin x^2$, $(\sin x)^2$?
- True or false: ${}_n C_r = {}_n C_{n-r}$

In each of these examples, although the question I am asking the class has only one answer, my understanding of my students' knowledge and misconceptions tells me the question could lead to (usually, but not always!) a quality debatable

moment. Having debate routines in place allows these unplanned moments to easily turn into a debate or a discussion. Some poking or prodding with questions like *What if we use negative numbers?* *What about fractions?* *Can we try to draw strange examples?* can get students questioning their initial solutions, and eventually we get a second opinion on the possible answer.

As another example, consider Figure 3.4. If you were to ask students to find the area of the triangle, that would be a closed question. Combining this closed question with student “mistakes” in Figure 3.5 leads to a debatable moment with: *Who has the best mistake?*

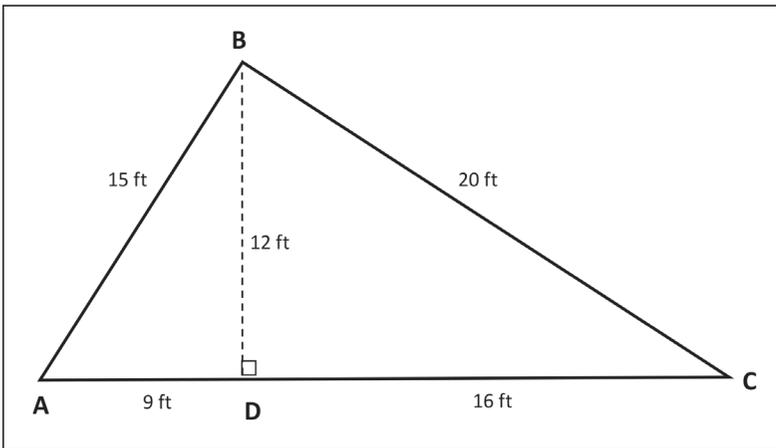


FIGURE 3.4

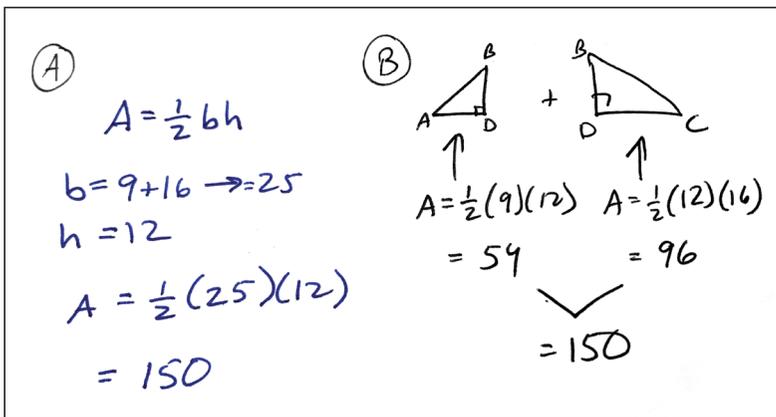


FIGURE 3.5

Notice that there actually are no mistakes here. However, knowledge of student misconceptions at this point lead me to believe this question would engender a debate. For example, we might hear students respond (mathematically incorrectly) to the question of the best mistake with:

Netta: I claim the first one has the best, and my warrant is because they used the wrong sides. You're supposed to use 15 and 20.

or

Roberta: I claim the second one is the best mistake, and my warrant is that you can't change the problem to two triangles.

These statements allow us as a class to discuss some misconceptions in finding the area of a triangle. The question is not technically debatable, but knowledge of students' confusions led to it becoming a debate activity.

So, please, forgive me if I continue to mix some closed questions in with truly debatable questions in my examples. In my class, I may allow a second opinion on a problem that only has one answer.

SO MANY QUESTIONS

In this chapter, we have talked about making questions debatable by using Best/Worst Starters; introducing “debate-y” words; framing questions as Always, Sometimes, Never; using students' mistakes or their own words; and debating misconceptions. Armed with these tools, let's take a question on multiplying integers, such as $3(-4) = \underline{\quad}$, and make it debatable.

Solve $3(-4) = \underline{\quad}$

- What is the best first step to solve this problem?
- What is the coolest way to model this problem?
- What is the easiest mistake you can make in solving this problem?
- Will there always, sometimes, or never be negative solutions?
- Which student had the best mistake? (See Figures 3.6A and 3.6B.)
- A student said, “When there's more than one negative, the answer is positive.” Is that true?
- Does multiplication always result in a larger number?

$$3(-4) = -1$$

FIGURE 3.6A

$$3(-4) = 12$$

FIGURE 3.6B

Similarly, we could also consider a question on the quadratic formula, such as solving for x in the equation: $x^2 + x = 12$.

Solve for x : $x^2 + x = 12$.

- What is the best first step in solving this problem?
- What is the easiest method for solving this equation?
- What is most common mistake someone can make when solving this problem?
- Will there always, sometimes, or never be two solutions?
- Which student had the best mistake? (See Figures 3.7A and 3.7B.)
- Let's debate: "Some equations have two real number answers. Some have two imaginary answers. It isn't possible to have one real and one imaginary answer."
- True or false: $x^2 + x - 12 = 0$, $0 = 12 - x^2 - x$, and $0 = 12 + x^2 - x$ are all the same.

Notice the variety of open and closed questions that could spark a discussion and debate in your math classroom. Those are just seven examples of ways to make the question more debatable. Can you think of a few more options?

$$x(x+1) = 12$$

$$x = 0, -1$$

FIGURE 3.7A

$$\begin{aligned}x^2 + x - 12 &= 0 \\x &= \frac{1 \pm \sqrt{1^2 - 4(1)(-12)}}{2} \\&= \frac{1 \pm \sqrt{1 - 48}}{2} \\&\text{imaginary!}\end{aligned}$$

FIGURE 3.7B

DEBATE EVERYTHING!

Every time I work with teachers, I say that any math problem can become debatable. I make this broad claim partly to be controversial when talking about a subject that many (especially people outside our profession) believe to be unambiguous. I also say it because I want to push teachers to focus a little less on the correct answer and a little more on students' abilities to discuss and debate the concepts and processes.

I always challenge teachers to stump me—to come up with a math question for students that I *cannot* turn into a debatable moment. Although I would happily accept being stumped by an idea, I have yet to hear a question that I could not turn into a debatable moment. I say this not to brag, but to encourage you to think more openly about the questions you ask, about what you see as a closed question. Let your mind be open to making statements more debatable, and if you get stuck, send me a message or a tweet (@cluzniak, #DebateMath). Let's help one another raise the bar for debating math.