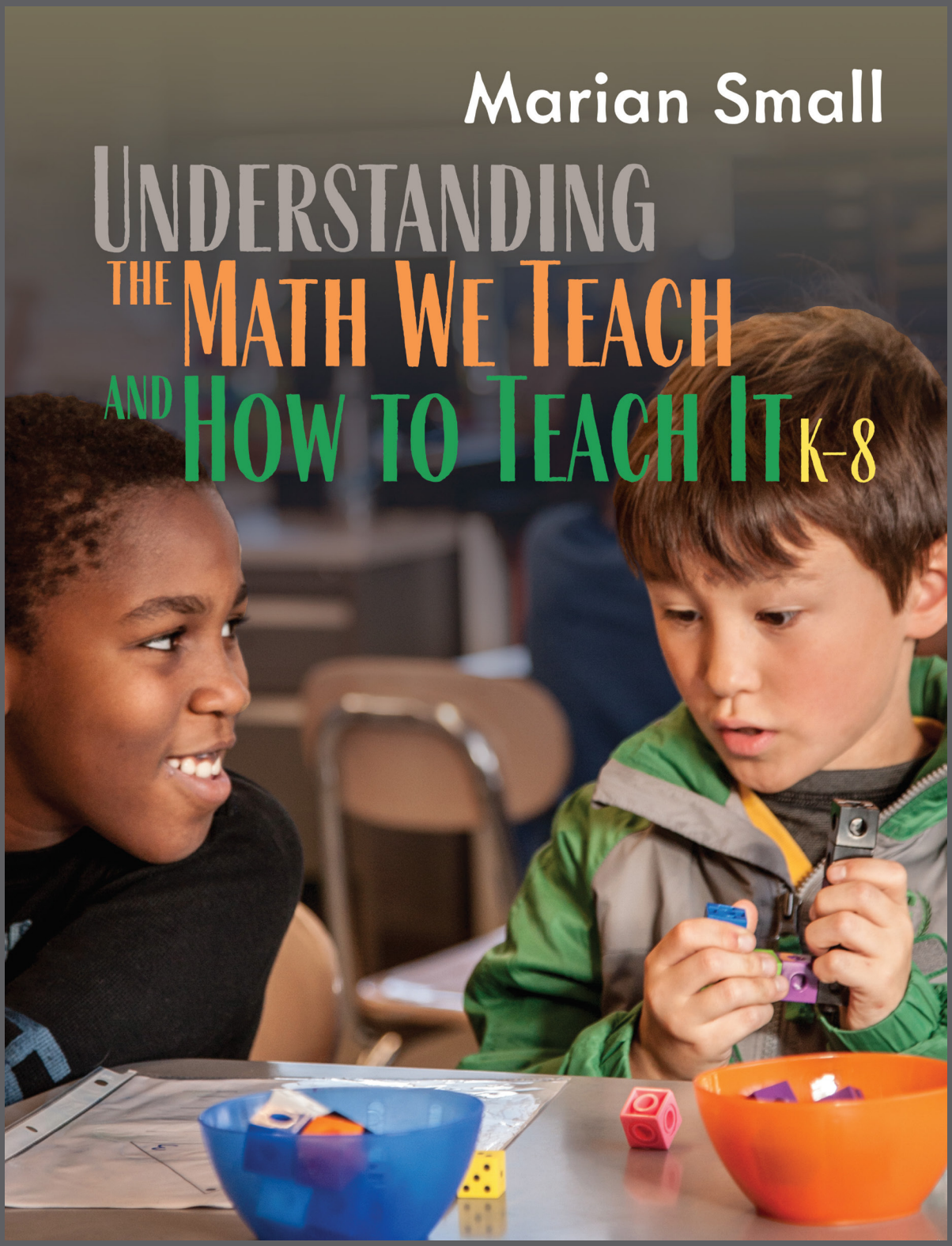
Study Guide



Marian Small

Understanding the Math We Teach and How to Teach It K–8

Professional development training for teachers provided by Stenhouse Publishers

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Contents

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Chapter 1
How Students Learn Math and What Math We Want Them to Learn

Chapter Summary

The main purpose of this chapter is to set the stage for the rest of the text by introducing the prospective or practicing teacher to the big issues in mathematics education. Many of the ideas are examined in more detail in subsequent chapters.

Key ideas:

1. There are divergent opinions on what mathematics should be learned at what grade levels.
2. There is a great deal of consistency, among mathematics educators, that mathematics should be learned by developing meaning rather than memorizing skills.

Before the Meeting

Ask participants to do the following:

1. Read Chapter 1.
2. Complete *Applying What You've Learned* question 10 or 11 (page 12) to interact with a student or another teacher (or both!) about the content of the chapter.
3. Jot down notes in response to the passage below, from page 4, and bring these notes to the meeting:

"In a constructivist classroom, it is through interactions with other students as well as with the teacher, and with the opportunity to articulate their own thoughts, that students are able to construct new mathematical knowledge. These classrooms are ones where varied approaches are expected, shared, and valued. The teacher's role is less about standing at the front talking and more about circulating, engaging in small conversations, facilitating, and gathering information to inform future instruction." (page 4)

During the Meeting

Sharing the Pre-work

Because this is your first meeting, it's important to set a positive tone and establish productive ways of working together. Discussing the responses to the excerpt on page 4 might be a good way to start. Teachers may naturally reflect on how different this picture is from their own memories as math students and articulating the differences can help frame their thinking.

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

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Do Some Math Together: Solve the Chapter Problem (page 1)

Throughout the book, there are many opportunities to do mathematics together, either with the chapter problems or the embedded activities. Make sure to emphasize that working on these problems together are valuable opportunities for colleagues to learn together, both about mathematical content and pedagogical content knowledge. Participants should expect to "not know" or be unsure sometimes and will be able to lean on one another to think problems through together.

There are many ways to think about the bike and car problem, and it may be productive to discuss solutions with your colleagues. For example, some colleagues might notice that the bike goes 28 mph slower than the car. The car needs to make up 112 miles. Since it goes 28 mph faster, it would take 112 miles ÷ 28 mph = 4 hours.

Other participants may have started by thinking, "The car needs to travel 112 miles at 50 mph to get to the bike's starting point. That would take about 2:14. But during that time, the bike has continued traveling. At 22 mph, in 2:14, the bike could cover another 49 miles. To drive that distance, the car needs almost another hour . . . " and so on, catching the car up to the bike in pieces.

There are other possible approaches as well. Did any participants make a representation or have a strategy for keeping track of their thinking that might be interesting to discuss?

It's important to clarify that there isn't a "right" way to approach this problem, or any of the other chapter problems you'll discuss as a group.

Possible Discussion Questions

1. This chapter models what it means to really understand 3×5.
   - Select other topics in K to 8 and come to a consensus about what it really means to understand those topics. For example, list four or five things that a student could say, write, or do to show a full understanding of what $28 \div 4$ means.
   - How could having teachers articulate what it really means to understand a particular concept help those teachers teach more effectively?
   - How might you convince someone that an over-focus on procedures might be detrimental to student learning?
2. In this chapter, there are several references to the importance of student autonomy in the classroom.
   - What do you think student autonomy might mean in a learning situation and why might it be valuable?
   - What evidence would you gather to show that student autonomy exists in a classroom?
   - How much autonomy do you see in classrooms you visit?
   - Does that autonomy increase or decrease as you go up the grades?

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3. A number of lenses through which people view mathematics are cited in the chapter.
   - List some different perspectives or lenses one might hold on what the essential focus of math teaching should be.
   - Which lens resonates most with you? Why?
   - How does that lens influence how you teach?
   - How does it influence your interactions with students and parents?
   - How did your own math teachers seem to view what mathematics is? What evidence do you have to support this? How is your perspective similar or different?
4. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 2.
2. Complete one of the *Applying What You've Learned* questions, 8, 9, or 10 (page 32) to interact with a student or another teacher (or both!) about the content of the chapter.
3. After reading about the "big ideas," select a soon-to-be-taught lesson and identify some big ideas in it. Participants should bring this lesson with them to the meeting.

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Chapter 2
Focusing Instruction on Big Ideas and Mathematical Processes

Chapter Summary

The main purpose of the chapter is to provide a "big picture" of math—what are the big content ideas and what are the important processes that span the grades and lead to the most enduring understandings.

Key ideas:

1. How mathematical processes are treated in various curriculum documents
2. How mathematical processes, big ideas, and organizers like proportional reasoning, algebraic reasoning, and spatial reasoning are different from traditional "topics" in math

Before the Meeting

Ask participants to do the following:

1. Read Chapter 2.
2. Complete one of the *Applying What You've Learned* questions, 8, 9, or 10 (page 32) to interact with a student or another teacher (or both!) about the content of the chapter.
3. After reading about the "big ideas," select a soon-to-be-taught lesson and identify some big ideas in it. Participants should bring this lesson with them to the meeting.

During the Meeting

Sharing the Pre-work

In partners or small groups, participants can share the lessons they brought and the big ideas they identified. There may be some disagreement or discussion about what counts as a "big idea." It might be productive to discuss these cases in the larger group. Invite participants to look for evidence supporting their arguments within Chapter 2.

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

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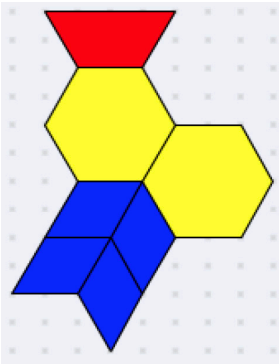
Do Some Math Together: Solve the Chapter Problems (page 15)

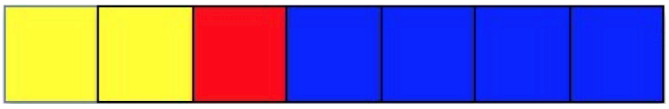
There are many solutions to both of these problems. For example, this would be one possible solution for the first problem:



What do you notice or wonder when you and your colleagues look at your designs? What similarities and differences do you see?

The fraction of the area that is yellow is $\frac{12}{23}$ in the design shown above. Did different designs yield different fractions for area?

For the second problem, this would be one possible solution:



Are there other solutions?

In the solution shown, the fraction of the area that is yellow is $\frac{2}{7}$. Did different designs yield different fractions?

What important ideas about proportional reasoning do you think these problems raise?

Possible Discussion Questions

1. Different standards advocate a different organization for mathematical processes.
   - Opening up your mind to all of the possibilities, which one do you think might be most helpful for teachers to consider when planning teaching?
   - What are some potential benefits of emphasizing processes as well as content when planning mathematics instruction?

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2. Which one or two big ideas in number do you think is (are) most important? Why? Why would it not be appropriate to call multiplication a big idea in number?
3. Choose a large topic, for example, fractions, and consider and discuss ways that multiple representation and visualization play a role in understanding that topic. How could thinking about the topic this way positively impact our instruction?
4. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 3.
2. Complete either *Applying What You've Learned* question 9 or 10 (pages 60–61) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Select a recent assessment, or plan an upcoming one, and bring it to the meeting. Look at the list of assessment data sources on page 40. Using this list, how would you characterize your assessment? If you have already used the assessment and have student work, observational notes, or other artifacts to share, bring some with you.

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Chapter 3

Assessment and Evaluation

Chapter Summary

This chapter focuses on the fundamental role assessment plays in instructional decision-making.

Key ideas:

1. The differences between assessment *for* learning, assessment *as* learning, and assessment *of* learning
2. There are many complex issues surrounding good assessment
3. There are many assessment strategies available to use

Before the Meeting

Ask participants to do the following:

1. Read Chapter 3.
2. Complete either *Applying What You've Learned* question 9 or 10 (pages 60–61) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Select a recent assessment, or plan an upcoming one, and bring it to the meeting. Look at the list of assessment data sources on page 40. Using this list, how would you characterize your assessment? If you have already used the assessment and have student work, observational notes, or other artifacts to share, bring some with you.

During the Meeting

Sharing the Pre-work

Ask participants to share the assessments they brought with them in pairs or small groups. Which type of assessment is it (page 40)? What did/could the teacher learn about student thinking by administering it?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 35)

How did participants approach this problem? Did some take a unit rate approach? Or draw on outside experience to make assumptions about the pricing of the big bag? Or use proportional reasoning?

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One way to think about the first problem is:

- The marbles in the small bag cost about 5¢ each. The marbles in the medium bag cost about 4.5¢ each. Since the marbles seem to cost less if you buy more, the marbles in the big bag might cost 4¢ each; so there should be about 260 marbles in the bag.

One possible argument for a solution to the second problem is:

- To have 300 marbles, it would be best to buy one large bag (that's 260 marbles) and 1 small bag (another 52 marbles). The total cost would be $13. If he bought 2 medium bags, it would cost too much.

Would this task be a suitable performance task? Why?

Possible Discussion Questions

1. On pages 38 and 39, there is a list of elements of a good assessment plan. Which do you believe are most essential? Least essential? Why?
2. How do assessment *for* learning and assessment *of* learning differ? Could the same task ever be suitable for both?
3. Some teachers feel that they can't afford the time to assess both processes and content. What advice would you give them?
4. One of the difficulties of using a significant amount of group work is the issue of individual evaluation. How could a teacher work around this problem?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 4
2. Encourage colleagues to complete either *Applying What You've Learned* question 9 or 10 (page 87) to interact with a student or a teacher (or both!) about the content of the chapter.
3. After reading the chapter, plan an upcoming lesson. What choices did you make with regards to lesson style, lesson strategies, types of collaboration, support, practice, and differentiation? Why did you make these choices?

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Chapter 4

Planning Instruction

Chapter Summary

This chapter emphasizes the importance of careful and thorough planning for the success of math instruction.

Key ideas:

1. The necessity of well-considered planning of instruction
2. The necessity of varied instruction
3. The necessity of differentiating instruction

Before the Meeting

Ask participants to:

1. Read Chapter 4.
2. Encourage colleagues to complete either *Applying What You've Learned* question 9 or 10 (page 87) to interact with a student or a teacher (or both!) about the content of the chapter.
3. After reading the chapter, plan an upcoming lesson. What choices did you make with regards to lesson style, lesson strategies, types of collaboration, support, practice, and differentiation? Why did you make these choices?

During the Meeting

Sharing the Pre-work

Ask participants to share their lesson plans in pairs or small groups, focusing especially on the decisions they wrestled with.

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solving the Chapter Problem (page 63)

This is an open-ended problem, with many possible solutions. How did participants grapple with it? Did some use a guess-and-check approach at first? Did different strategies emerge as they kept working?

Three possible words are *pacer*, *glare*, and *dart*.

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It might be enjoyable to think of extension questions that springboard off the first question, such as:

- What's the shortest/longest word we can think of?
- What if we looked for words worth 100 instead of 43? How might our approaches differ?
- What other questions can we generate?
- The chapter opens with a discussion of math anxiety. It might be interesting to ask if participants felt more or less anxiety with this problem when they realized there were multiple solutions and multiple approaches?

Possible Discussion Questions

1. What are some actions you think teachers and parents can take to minimize math anxiety in children?
2. In the previous chapters, we talked about the big ideas. Sometimes, when planning, teachers shift their attention to what students are *doing* more than what students are *learning*. How can we maintain focus on the "big ideas" while planning the details of instruction, tasks, and activities?
3. Many educators advocate the three-part lesson, sometimes called "activation, problem, consolidation" or "launch, explore, synthesize." This is a different model than what is sometimes called the "I do, we do, you do," which involves a demonstration by the teacher, guided practice, and independent practice. In the three-part lesson, the launch is brief, the students are working on a rich problem or problems for much of the lesson, and the synthesis discussion is very important.
   - What are the most important things to accomplish in the introduction to the lesson?
   - What should the teacher be doing while the students are working on the problem?
   - What are the most important things to accomplish in the close of the lesson?
4. The chapter addresses the importance of a varied lesson style, but some would argue for consistency. What is your position on this issue and why?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 5.
2. Complete either *Applying What You've Learned* question 8 or 9 (page 107) to interact with a student or a teacher about the content of the chapter. In this case, it's strongly preferable to do both, and especially important to listen to a young child count a variety of objects.
3. Choose a picture book that has productive opportunities to count in ways that support enjoyment of a story, such as *Baby Goes to Market*. Bring the book to the meeting. For contrast, consider also bringing a counting book that you are less impressed with.

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Chapter 5

Early Number

Chapter Summary

The focus of this chapter is the beginning of number development. The emphasis is on numbers to 20, but mostly on numbers to 10. Once students can count meaningfully, have an understanding that each number can be represented many ways, and can relate numbers to each other, they are much more likely to succeed in their later number work.

Key ideas:

1. The complexity of the counting process
2. The importance of ensuring that students represent numbers in multiple ways and relate numbers to each other

Before the Meeting

Ask participants to do the following:

1. Read Chapter 5.
2. Complete either *Applying What You've Learned* question 8 or 9 (page 107) to interact with a student or a teacher about the content of the chapter. In this case, it's strongly preferable to do both, and especially important to listen to a young child count a variety of objects.
3. Choose a picture book that has productive opportunities to count in ways that support enjoyment of a story, such as *Baby Goes to Market*. Bring the book to the meeting. For contrast, consider also bringing a counting book that you are less impressed with.

During the Meeting

Sharing the Pre-work

Invite participants to take a book walk, flipping through and thinking about what makes for engaging counting books. What criteria should we apply in choosing counting books to use with young students?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting. In this case, make sure teachers have time to talk about what they learned when observing young children counting.

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Do Some Math Together: Solve the Chapter Problem (page 89)

How did colleagues approach this problem? How might students of different ages approach this problem? Some examples of responses:

Alike: Both numbers are even, less than 10, greater than 1, one away from 7, and the numerals are round.

Different: Only one is divisible by 3, only one is divisible by 4, only one has a square that has the same ones digit as the original number, and only one is a factor of 12.

Participants might notice that there's nothing to "solve" here, exactly. How does that change the nature of the question?

Possible Discussion Questions

1. What is the difference between rote counting and meaningful counting?
   - What is the value of each kind of counting?
   - What kinds of activities and experiences help students develop rote counting skills?
   - What kinds of activities and experiences help students develop meaningful counting skills?
2. The book says, "Counting is fundamental to further number work." What does this mean? Think about the role of counting across various mathematical topics and grade bands: K–2, 3–5, and 6–8.
3. Parents of kindergarten and first-grade children often point out how high their child can count as a sign of mastery of grade-level content. How could you respond to these parents to help them understand the important and robust counting work going on in the primary grade and what it means for their child?
4. What is the value of using 5 and 10 as benchmarks against which other numbers can be compared?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 6.
2. Complete either *Applying What You've Learned* question 9 or 10 (page 134) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look at the Addition and Subtraction Situations in the chart on pages 112 and 113. Choose one row of the chart and write your own story problems for each problem type in that row. Bring these problems to the meeting.

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Chapter 6

Early Operations

Chapter Summary

The main purpose of this chapter is to provide support for teachers to help students to develop a fuller understanding of the four operations and to help them make students' acquisition of knowledge about the four operations meaningful and conceptual.

Key ideas:

1. Different meanings for the operations
2. How the four operations are related

Before the Meeting

Ask participants to do the following:

1. Read Chapter 6.
2. Complete either *Applying What You've Learned* question 9 or 10 (page 134) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look at the Addition and Subtraction Situations in the chart on pages 112 and 113. Choose one row of the chart and write your own story problems for each problem type in that row. Bring these problems to the meeting.

During the Meeting (cubes or counters needed)

Sharing the Pre-work

Ask participants to partner up and trade problems that they've written. Each teacher should use cubes, chips, or counters to model solving each problem their partner created, talking about the actions (or non-actions) involved in each problem as they go. Make sure participants actually move the cubes, rather than just imagine moving them. What do participants notice about their thinking during different problem types?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

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Do Some Math Together: Solve the Chapter Problem (page 109)

Ask colleagues to share out some possible story problems using the words given.

For example: Some kids shared equally most of the 24 bananas they had. If each kid got less than 3 bananas, how many kids might have shared them?

To solve it with addition instead of division, you could solve the problem by adding up a bunch of 2s to get up to 24. Because $2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 + 2 = 24$, it could have been up to 12 kids.

Many teachers focus on teaching "keywords" to students when solving word problems, such as *"in all* means add." What do you think about this strategy after writing the banana problem and acting out different problem types?

Possible Discussion Questions

1. How are the four operations—addition, subtraction, multiplication, and division—related?
   - Why is it important for students to understand the relationships between these operations?
2. Choose two of the operations. How would you help students recognize the relationship between those two operations? Why do you think that your approach might be effective?
3. As novices, it can be easy for students to make generalizations such as "subtraction means take away" or "multiplication means repeated addition."
   - Why are these generalizations problematic, especially if students carry them into later grades?
   - What do we need to do as educators to help students develop a more robust understanding of the meanings of each operation?
4. Brainstorm all the different ways you can think of to model the expression 9×4.
   - What is similar and different about each model?
   - Why is it important for students to spend time creating and making sense of a variety of models as they're learning about each of the operations?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 7.
2. Complete one of the *Applying What You've Learned* questions 8, 9, or 10 (page 148) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Bring a current math textbook for Grades 1 or 3. Briefly review the lessons on addition (Grade 1) or multiplication (Grade 3).

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Chapter 7

Developing Fact Fluency

Chapter Summary

The main purpose of the chapter is to provide support for teachers to help students develop number fact fluency.

Key idea:

1. Strategies students can and should use to learn addition, subtraction, multiplication, and division facts

Before the Meeting

Ask participants to do the following:

1. Read Chapter 7.
2. Complete one of the *Applying What You've Learned* questions 8, 9, or 10 (page 148) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Bring a current math textbook for Grades 1 or 3. Briefly review the lessons on addition (Grade 1) or multiplication (Grade 3).

During the Meeting (blank 0–9 addition tables needed)

Sharing the Pre-work

Ask participants to partner up with someone and look through their texts together to see which addition (Grade 1) or multiplication (Grade 3) strategies are emphasized in the text. Pay particular attention to which strategy is introduced first. Why might that make sense, or not?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 137)

How did participants approach this task? What did they try first? What patterns did they observe? How did they make use of patterns to help answer the question?

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+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

Each diagonal running southwest to northeast on the table has all the same numbers on it. Nine appears most often in the table because the diagonal containing 9s is the longest.

How could you have predicted the answer without completing the table?

In Chapter 5, participants read about the importance of multiple representations of numbers. How can that foundational work support this later work learning these addition combinations?

Possible Discussion Questions

1. What is fact fluency? Why is it important for students to develop fact fluency?
2. The book says, "An instantaneous response is not as much the goal, as a reasonably quick, but well understood, response to a math fact question." What does this mean? What are some examples of what a "reasonably quick, but well understood, response" might look like or sound like?
3. A parent asks you whether it is a good idea to use flashcards for fact recall for her Grade 1 child. What would your advice be?
4. Often fact fluency is synonymous with timed tests. Think back to Chapter 4 on assessment. What are some different ways a teacher can assess students' fact fluency? Would each assessment type reveal the same information about student's fact fluency?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 8.
2. Complete either *Applying What You've Learned* question 9 or 10 (page 170) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look for examples of large numbers, in your life, in your community, or in the world. Bring a few examples to share.

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Chapter 8

Representing Larger Whole Numbers

Chapter Summary

The main purpose of the chapter is to explore the complexities of modelling greater whole numbers in a meaningful way for students as well as concepts focused on properties of numbers.

Key ideas:

1. The principles students must understand in representing and comparing large whole numbers
2. The value of proportional materials in representing large numbers
3. The potential for number theory as a tool for student investigation of number

Before the Meeting

Ask participants to do the following:

1. Read Chapter 8.
2. Complete either *Applying What You've Learned* question 9 or 10 (page 170) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look for examples of large numbers, in your life, in your community, or in the world. Bring a few examples to share.

During the Meeting

Sharing the Pre-work

Put participants into small groups to share their examples of large numbers. Afterward, ask participants to share out large numbers that were particularly interesting. Why is it important to look for and share real-life examples of large numbers with students? What is the value in looking for examples of large numbers within the school or the students' communities?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 149)

How did participants approach this problem? Did anyone try solving a smaller problem first? For example, finding the least whole number that is divisible by 2 and 3? By 2, 3, and 4? By 2, 3, 4, and 5? What other strategies did participants try?

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The least whole number is 2,520. If you multiply $2 \times 3 \times 2 \times 5 \times 7 \times 2 \times 3$, you get 2,520. It is divisible by each of the listed numbers (2, 3, 4, 5, 6, 7, 8, 9, and 10) because its factors are there. It is the least number since only necessary factors were included.

Which concept in the chapter does this problem relate to? If it didn't naturally come up in conversation already, this is a great time to help participants recognize these are all prime numbers that can be used to make all the factors in the list:

- $2 \times (3 \times 2 \times 5 \times 7 \times 2 \times 3)$
- $3 \times (2 \times 2 \times 5 \times 7 \times 2 \times 3)$
- $(2 \times 2) \times (3 \times 5 \times 7 \times 2 \times 3)$
- $5 \times (2 \times 3 \times 2 \times 7 \times 2 \times 3)$
- $(2 \times 3) \times (2 \times 5 \times 7 \times 2 \times 3)$
- $7 \times (2 \times 3 \times 2 \times 5 \times 2 \times 3)$
- $(2 \times 2 \times 2) \times (3 \times 5 \times 7 \times 3)$
- $(3 \times 3) \times (2 \times 2 \times 5 \times 7 \times 2)$
- $(2 \times 5) \times (3 \times 2 \times 7 \times 2 \times 3)$

Possible Discussion Questions

1. What are different ways that students can meaningfully represent large numbers? What benchmarks can help students develop a deeper understanding of these numbers?
2. What is the difference between proportional and nonproportional materials for representing large numbers? How do these differences influence the materials teachers choose to include in a lesson?
3. What big idea(s) do we want students to develop around place value, especially as students are representing and working with larger and larger numbers?
4. Some people would argue that knowing about prime numbers is an unnecessary aspect of the math curriculum. How would you respond?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 9.
2. Complete either *Applying What You've Learned* question 11 or 12 (page 207) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Bring a current math textbook for Grade 1, 2, 3, or 4. Select a lesson about subtracting or adding multidigit numbers. Do you think the lesson is well-constructed? Be prepared to share reasons why or why not.

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Chapter 9

Estimation and Calculation Strategies with Larger Whole Numbers

Chapter Summary

The main purpose of the chapter is to describe the variety of valuable approaches there are for children to address the four basic operations of adding, subtracting, multiplying, and dividing with multi-digit whole numbers.

Key ideas:

1. The value of alternate algorithms
2. The idea that any algorithm should be explainable

Before the Meeting

Ask participants to do the following:

1. Read Chapter 9.
2. Complete either *Applying What You've Learned* question 11 or 12 (page 207) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Bring a current math textbook for Grade 1, 2, 3, or 4. Select a lesson about subtracting or adding multidigit numbers. Do you think the lesson is well-constructed? Be prepared to share reasons why or why not.

During the Meeting

Sharing the Pre-work

Ask participants to partner up with someone to share the lesson they've chosen. They should explain why they think the lesson is or is not well constructed. What changes could be made to the lesson to make it better? How can we determine when to modify a lesson versus when to abandon it to create or seek out an alternative?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

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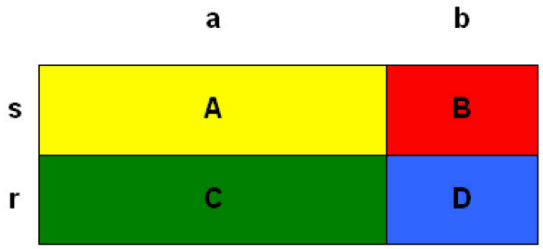
Do Some Math Together: Solve the Chapter Problem (page 173)

What is your solution to the chapter problem? Can you explain why your "rule" works?

$A \times D = B \times C$

To show why it happens, you might describe each part of the length and each part of the width:



A = as B = bs C = ar D = br

So $A \times D$ = asbr and $B \times C$ = bsar.

The same four values are multiplied in each case, so the products of the areas are equal.

Possible Discussion Questions

1. To calculate 5,002 – 3,189, a teacher suggested decomposing 3,189 into 3 and 3,186. First, she took away 3 from 5,002, then she took away 3,186.
   - Why might she do this?
   - Do you think it is a good idea to make that suggestion?
   - How does this method compare to using the standard US algorithm for subtraction?
2. What is the value in allowing students opportunities to invent their own procedures, including mental math strategies? How does this influence planning at the year, unit, and/or lesson level?
3. Many people argue that base-ten blocks are an essential manipulative to explain the algorithms for the four operations. Do you agree? Why or why not?
4. Look at the charts of estimation strategies on page 175 and page 191.
   - Should estimation always involve rounding to the nearest ten, hundred, thousand, etc.? Why or why not?
   - Why is it important for students to develop multiple strategies for estimating?
   - How much time should be spent on estimation as students are learning how to calculate with larger numbers?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

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After the Meeting:

Ask participants to do the following:

1. Read Chapter 10.
2. Complete either *Applying What You've Learned* question 11 or 12 (page 244) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Choose a fraction greater than 0 and less than 1. Make a small poster representing your fraction in a variety of different ways. Bring your poster to the meeting.

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Chapter 10

Fractions

Chapter Summary

The main purpose of the chapter is to provide a firm foundation for dealing with the complexity of teaching about fraction concepts and fraction operations. The focus is on helping teachers teach in a meaningful way through the use of many manipulatives and many fraction meanings, rather than concentrating on procedural understanding.

Key ideas:

1. Some of the stumbling blocks for students in learning about fractions and ways around them
2. The critical importance of building understanding of fractions using concrete materials and pictorial representations and an understanding that a fraction is a single number

Before the Meeting:

Ask participants to do the following:

1. Read Chapter 10.
2. Complete either *Applying What You've Learned* question 11 or 12 (page 244) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Choose a fraction greater than 0 and less than 1. Make a small poster representing your fraction in a variety of different ways. Bring your poster to the meeting.

During the Meeting

Sharing the Pre-work

Have participants get into small groups to share out their posters. What is similar and different between the posters in each group? Why is it important for students to represent numbers using multiple representations? How does this relate to the work students do in the early grades learning about numbers to 10 and 20?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 209)

What understandings about fractions did you need to use to help you tackle this problem? What representation(s) did you use to help you make sense of the problem?

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One strategy is to rewrite both fractions so they have the same denominator.

$\frac{2}{7} = \frac{24}{84}$ and $\frac{5}{12} = \frac{35}{84}$

The factors of 84 are $2 \times 2 \times 3 \times 7$.

The smallest denominator you can try first is 2. To simplify to a denominator of 2, the numerator has to be $2 \times 3 \times 7$ or 42. $\frac{42}{84}$ is not between $\frac{2}{7}$ and $\frac{5}{12}$, so that won't work.

The next smallest denominator you can try is 3. In order to simplify to a denominator of 3, the numerator has to be $2 \times 2 \times 7$, or 28. $\frac{24}{84} < \frac{28}{84} < \frac{35}{84}$, and $\frac{28}{84}$ is equivalent to $\frac{1}{3}$.

Another strategy is to guess and check. Both fractions are less than $\frac{1}{2}$ so the lowest possible denominator will not be 2. Next you can compare both fractions to $\frac{1}{3}$.

$\frac{1}{3} = \frac{2}{6}$ and $\frac{2}{7} < \frac{2}{6}$, so $\frac{2}{7} < \frac{1}{3}$.

$\frac{1}{3} = \frac{4}{12}$ and $\frac{4}{12} < \frac{5}{12}$, so $\frac{1}{3} < \frac{5}{12}$.

$\frac{2}{7} < \frac{1}{3} < \frac{5}{12}$, so the fraction with the lowest possible denominator is $\frac{1}{3}$.

Possible Discussion Questions

1. We often focus on the part-whole meaning for fractions. Make a case for why the other meanings are also important to use.
2. How would you help a student understand why the whole needs to be considered when comparing two fractions?
3. A big idea in fractions is that "every fraction can be renamed in an infinite number of ways." What is the significance of this big idea? How does it connect to how students compare and compute with fractions?
4. Consider each of the four operations. What connections can you make between each operation when it's carried out with whole numbers and when carried out with fractions? Why is it important to make these connections explicit for students?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

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After the Meeting

Ask participants to do the following:

1. Read Chapter 11.
2. Complete either *Applying What You've Learned* question 8 or 9 (page 268) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look for examples of decimal numbers in your life, in your community, or in the world. Bring a few examples to share.

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Chapter 11

Decimals

Chapter Summary

The main purpose of the chapter is to explore options available in developing both strong links to fraction concepts and strong links to whole number concepts when teaching decimals to students.

Key ideas:

1. The language that is appropriate to use when explaining how to operate with, describe, or compare decimals
2. How to model decimals and decimal operations to make sense of how to compare two decimals or perform a calculation involving two or more decimals.

Before the Meeting

Ask participants to do the following:

1. Read Chapter 11.
2. Complete either *Applying What You've Learned* question 8 or 9 (page 268) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look for examples of decimal numbers in your life, in your community, or in the world. Bring a few examples to share.

During the Meeting (square tiles and/or grid paper needed)

Sharing the Pre-work

Put participants into small groups to share their examples of decimal numbers. Afterward, ask participants to share out decimal numbers that were particularly interesting. How could these real-life examples be incorporated into a unit on decimals or decimal operations?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

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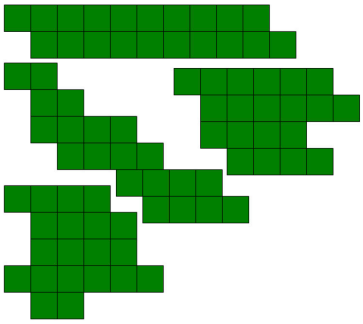
Do Some Math Together: Solve the Chapter Problem (page 247)

Do a gallery walk so participants can see their classmates' designs. For example:



What would a student need to understand about decimals to solve this problem?

What other math concepts could be tied in with this task?

Possible Discussion Questions

1. Decimals straddle the concepts of place value and fractions.
   - What ideas about place value do you think students should understand before you introduce decimals? Why?
   - What ideas about fractions do you think students should understand before you introduce decimals? Why?
2. Some people believe that it confuses students to use base-ten blocks as decimal representations when they had previously been used for whole numbers. How would you (or, would you) argue that they are still an important manipulative for understanding and carrying out decimal operations?
3. Look at Student Response 1 and Student Response 2 on page 267.
   - What do you notice and wonder about their responses?What strengths can you identify in their work?
   - What misconceptions can you identify in their work?
   - How would you intervene?
4. Some teachers believe the best way to approach a question like 4 – 1.23 is to have students set it up as 4.00 – 1.23, and then regroup using the standard US algorithm. Argue why it might be a good idea, and also argue why it might not.
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

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After the Meeting

Ask participants to do the following:

1. Read Chapter 12.
2. Complete either *Applying What You've Learned* question 8 or 9 (page 291) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Solve the following problem in two to three different ways. Bring your solutions to the meeting:
   - 72% of students in a school participated in the food drive. If 198 students participated, how many students are in the school?

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Chapter 12

Ratio and Proportion

Chapter Summary

The main purpose of the chapter is to help the reader see the connections between working with ratios, rates, percents, fractions, and decimals so that they can foster their students' ability to think proportionally.

Key ideas:

1. The power of ratio tables, graphs, and other visuals to solve ratio, percent, or rate problems
2. The importance of understanding equivalent ratios to solve ratio, percent, or rate problems
3. That there are always many ways to solve a ratio, percent, or rate problem

Before the Meeting

Ask participants to do the following:

1. Read Chapter 12.
2. Complete either *Applying What You've Learned* question 8 or 9 (page 291) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Solve the following problem in two to three different ways. Bring your solutions to the meeting:
   - 72% of students in a school participated in the food drive. If 198 students participated, how many students are in the school?

During the Meeting

Sharing the Pre-work

Ask participants to partner up with someone to share their solutions to the problem they solved before the meeting. Which of the ways they solved the problem (or saw how someone else solved the problem) do they personally find the most comfortable? Why?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 269)

This problem was ambiguous because it is not definitive about how long a lifetime is. What are some advantages and disadvantages of this sort of ambiguity?

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1. If you estimate that a person lives about 75 years, on average, then you would multiply $75 \times 365.25 \times 70$. Since an estimate makes sense, you might multiply $80 \times 350 \times 70$ to get $70 \times 28{,}000 = 1{,}960{,}000$, or about 2 million scalp hairs. This assumes that the rate is pretty constant (which it probably isn't).

2. 2 million = 20 hundred thousand, so 2 million ÷ 100,000 = 20; the equivalent to 20 heads of hair are lost.

Were you surprised by the solution to the problem? Why or why not?

Possible Discussion Questions

1. Look at the image of the parking lots on page 274. Students could be asked to identify which parking lot is "more full." Imagine a student says, "Lot 3 is more full because it only has 16 empty spaces, but lot 4 has 24 empty spaces."
   - What does this answer reveal to you about how this student is thinking about the question and the parking lots?
   - What kinds of activities and experiences might help this student so they can shift toward using proportional thinking with questions like this?
2. It was mentioned in the chapter that there is some informal proportional thinking that happens in grades K–5. What are some examples? How are they really about proportional thinking?
3. The Assessing Student Understanding section suggests the following open question: 72 is _____ % of _____ . Look at the Student Response to this open question on page 290.
   - What do you notice and wonder about this response?
   - What does it show that the student understands?
   - How was this question useful in assessing this student's understanding?
4. As students are learning about fractions, it is important for them to know what the whole is.
   - Why is this also an important idea when students are learning about percents?
   - What errors and misconceptions might arise when students do not consider the whole when working with percents?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 13.
2. Complete either *Applying What You've Learned* question 8 or 9 (page 291) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look for examples of negative numbers in your life, in your community, or in the world. Bring a few examples to share.

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Chapter 13

Extending the Number System to Negative and Irrational Numbers

Chapter Summary

The main purpose of the chapter is to provide approaches to meaningfully extend what students know about whole numbers to integers and what they know about fractions to rational and irrational numbers.

Key ideas:

1. The value of alternative representations for integers
2. The language to apply when discussing integer computation
3. The difference between rational and irrational numbers

Before the Meeting

Ask participants to do the following:

1. Read Chapter 13.
2. Complete either *Applying What You've Learned* question 8 or 9 (pages 313–314) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look for examples of negative numbers in your life, in your community, or in the world. Bring a few examples to share.

During the Meeting (integer tiles or two-color counters needed)

Sharing the Pre-work

Put participants into small groups to share their examples of negative numbers. Afterward, ask participants to share out negative numbers that were particularly interesting. How did looking for examples of negative numbers compare to looking for examples of large whole numbers? Could any of the examples shared be particularly useful for introducing any of the integer operations?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 292)

Which participants used tiles or counters to help make sense of the problem? How did they use them? What did these materials help participants see or understand that may not have been as obvious with symbols alone?

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Can –10 be a solution to this problem? Why or why not?

The only possible integers are (0), –3, –6, –9, –12, etc., i.e., negative multiples of 3.

How could you revise this problem so that the answer is negative multiples of 2? Positive multiples of 5?

Possible Discussion Questions

1. What do you see as the advantages and what do you see as the disadvantages of insisting that students use the + sign in describing positive integers?
2. Analyze the Student Response on page 313.
   - What do you notice and wonder about this student's work?
   - What feedback would you give this student?
   - How can we show that the zero property is fundamental to the other three operations as well?
3. What problems might you suggest to students to help them practice the various skills and concepts around powers in an interesting way?
4. Would a student need a set of rules about the order of operations to solve story problems involving a lot of computations, or is it just for complicated expressions involving only numbers?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 14.
2. Complete either *Applying What You've Learned* question 8 or 9 (page 350) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look for repeating patterns in the world around you. They can be found in nature, such as in flowers, gardens, and leaves, or in man-made products like clothing. Bring in physical examples, photos, and/or sketches to share.

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Chapter 14

Patterns and Algebra

Chapter Summary

The main purpose of the chapter is to provide background on the development of pattern and algebra concepts in students and the link that can and should be made between pattern thinking and algebraic thinking.

Key ideas:

1. How patterns and algebra are linked
2. The idea that without a pattern rule, there is no way to know how a pattern is continued
3. The critical role of pattern in learning other mathematical strands
4. That algebra is about generalization and about describing relationships
5. That there is more than one way to solve an equation or discern a relationship between two variables

Before the Meeting

Ask participants to do the following:

1. Read Chapter 14.
2. Complete either *Applying What You've Learned* question 8 or 9 (page 350) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Look for repeating patterns in the world around you. They can be found in nature, such as in flowers, gardens, and leaves, or in man-made products like clothing. Bring in physical examples, photos, and/or sketches to share.

During the Meeting

Sharing the Pre-work

Put participants into small groups to share their examples of repeating patterns. After participants have shared in small groups, ask if anyone would like to share out a particularly interesting pattern. Was it easy or difficult to find examples of patterns? What are the benefits to having students look for patterns around the school, at home, and in their community?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

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Do Some Math Together: Solve the Chapter Problem (page 315)

What problem solving strategy did participants use to solve the problem? Who made a model? Who drew a diagram? Who used a pattern?

In a group of 20 students, there would be 190 handshakes.

The first person shakes 19 hands; the next one 18 more hands; the next one 17 more hands; etc. The total number of handshakes is $1 + 2 + \ldots + 18 + 19$.

Notice that if $1 + 2 + \ldots + 18 + 19$ is added to $19 + 18 + \ldots + 2 + 1$ and the numbers are added vertically (as shown below), there are 19 pairs of 20.

$$\begin{array}{l} 1 + 2 + \ldots + 18 + 19 \\ \underline{19 + 18 + \ldots + 2 + 1} \\ 20 + 20 + \ldots + 20 + 20 \end{array}$$

This is twice as much as is desired, so the sum is 19×10, or 190.

How could this problem enhance a student's understanding of pattern?

Possible Discussion Questions

1. Look at the two patterns at the bottom of page 349. Why do you think the first pattern might be easier for a student to extend than the second?
2. A pattern begins 2, 5, … How could it continue? Let participants share a few different examples.Look at the Student Response to this same task on page 349.
    - What do you notice and wonder about this student's response?
    - How could this student's response be used to enhance your instruction?
3. A big idea that recurs throughout the book is the importance of creating and analyzing multiple representations of mathematical concepts. How does this big idea connect to learning about patterns and algebra?
    - What are the different representations of the 5, 8, 11, 14, 17, … pattern shown on pages 327 to 329?
    - What do each of these representations make more or less obvious about the pattern?
4. The introduction of letter variables is difficult for students. What are some strategies you think would be helpful for them to deal better with letter variables?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

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After the Meeting

Ask participants to do the following:

1. Read Chapter 15.
2. Complete one of the *Applying What You've Learned* questions, 9, 10, or 11 (page 398) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Choose a picture book that has productive opportunities to identify and discuss shapes, such as *The Greedy Triangle*. Bring the book to the meeting. For contrast, consider also bringing a shape book that you are less impressed with.

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Chapter 15

3-D and 2-D Shapes

Chapter Summary

The main purpose of the chapter is to explore those properties of 2-D and 3-D shapes that are most likely to be studied by K–8 students.

Key ideas:

1. How important it is to consider shapes broadly and explore many aspects of them
2. The importance of exploring 3-D and not just 2-D shapes
3. The important role of measurement in studying geometry

Before the Meeting

Ask participants to do the following:

1. Read Chapter 15.
2. Complete one of the *Applying What You've Learned* questions, 9, 10, or 11 (page 398) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Choose a picture book that has productive opportunities to identify and discuss shapes, such as *The Greedy Triangle*. Bring the book to the meeting. For contrast, consider also bringing a shape book that you are less impressed with.

During the Meeting (small mirrors needed)

Sharing the Pre-work

Invite participants to take a book walk, flipping through and thinking about what makes for engaging shape books. What criteria should we apply in choosing shape books to use with young students?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

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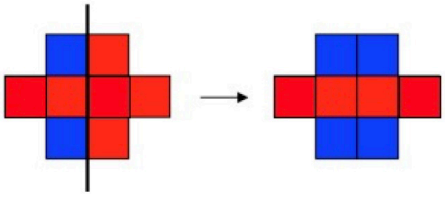
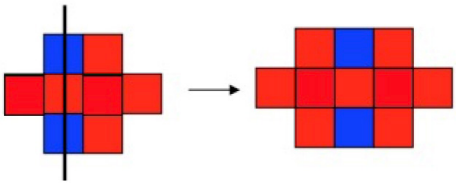
Do Some Math Together: Solve the Chapter Problem (page 353)

Would you assign this problem to a student to solve without providing a mirror? Why or why not?

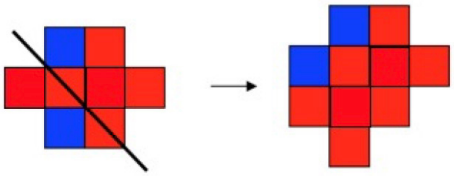
First one: I know because I saw that the left half of the shape stayed the way it was, but the right half didn't. You would have to look through the mirror from the left side.



Second one: I know because I saw that the right half of the shape stayed the way it was and so did the middle, but the left half didn't. By putting the mirror on the center of the middle, I could make that happen if I looked through the mirror from the right side.



Third one: I noticed that the top right of the shape remained, but not the bottom left. By putting the mirror on the diagonal and looking from the top right, it looks like the design shown.



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Possible Discussion Questions

1. A distinction is made between properties and attributes of shapes. Why is that distinction a useful one?
2. Many primary teachers prefer to begin with 2-D geometry rather than 3-D geometry because they believe it to be simpler and more accessible. How could you convince them that starting with 3-D work in geometry might be more appropriate?
3. How much emphasis do you feel should be placed on geometry vocabulary?
4. One of the key ideas in geometry is that every shape can be viewed as a part of or a combination of other shapes. Why might this be a useful unifying idea in the study of geometry?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 16.
2. Complete either *Applying What You've Learned* question 9 or 10 (page 424) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Search online for two "fun" activities that students could do with a Mira. Bring copies of the activities to the meeting.

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Chapter 16

Location and Movement

Chapter Summary

The main purpose of the chapter is to provide background on those aspects of geometry that relate to position, specifically the use of vocabulary and grids to describe position and the effects of transformations.

Key ideas:

1. The difference between different types of grids
2. How to distinguish between the Euclidean transformations

Before the Meeting

Ask participants to do the following:

1. Read Chapter 16.
2. Complete either *Applying What You've Learned* question 9 or 10 (page 424) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Search online for two "fun" activities that students could do with a Mira. Bring copies of the activities to the meeting.

During the Meeting (grid paper needed)

Sharing the Pre-work

Ask participants to partner up with someone and share the activities they brought. For each activity identify what mathematics students are learning and discuss whether that activity is a good way to approach the math.

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 401)

What information about transformations did you use to solve this problem?

For example, the coordinates could have been (2, 5), (10, 5), and (10, 11).

To get a solution, rotate the triangle a quarter turn clockwise around the point (2, 5). The new vertices are (2, 5), (2, –3), and (8, –3). Then reflect in the y–axis.

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Could your solutions for the starting points of the triangle still work if the triangle had been reflected and then rotated?

Possible Discussion Questions

1. Properties of shapes are often the focus of attention in geometry. Why is development of skills in describing and predicting location also an important aspect of spatial sense?
2. There is a continuum of simple to much more sophisticated systems to describe location. Even the youngest elementary school students can use a simple system.
   - A simple system introduced to young students is to teach them to use positional vocabulary. Which terms would you use earliest, and which would you delay? Why?
   - What kinds of activities and experiences can help older students transition toward more sophisticated systems to describe location?
3. Why is it especially important to include observation of students working when assessing student understanding of the skills and concepts in this chapter?
4. How could computer programming using a kid-friendly program such as Scratch help students think about position and space?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 17.
2. Complete either *Applying What You've Learned* question 10 or 11 (page 475) to interact with a student or a teacher (or both!) about the content of the chapter.
3. In a grade 2 or 3 textbook, locate a lesson on measuring length and a lesson on measuring area. Bring the textbook with you to the meeting.

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Chapter 17

The Nature of Measurement, with a Focus on Length and Area

Chapter Summary

The main purpose of the chapter is to explore general measurement concepts. This chapter includes an emphasis on length and area measurement because of the prominence of these two types of measurements in the curriculum.

Key ideas:

1. The measurement principles discussed at the start of the chapter
2. The three stages of measurement instruction and why they matter
3. Some of the common difficulties students have when measuring length and area and how these might be addressed

Before the Meeting:

Ask participants to do the following:

1. Read Chapter 17.
2. Complete either *Applying What You've Learned* question 10 or 11 (page 475) to interact with a student or a teacher (or both!) about the content of the chapter.
3. In a grade 2 or 3 textbook, locate a lesson on measuring length and a lesson on measuring area. Bring the textbook with you to the meeting.

During the Meeting

Sharing the Pre-work

Ask participants to partner up with someone and share the lessons they've chosen. Identify whether each lesson is strong or weak. Identify its strengths or deficits. How can the weak lessons be modified so they become stronger lessons?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 427)

What problem-solving strategy did participants use to solve the chapter problem?

The dimensions are 9 inches, 12 inches, and 15 inches.

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The area is $\frac{9 \times 12}{2} = \frac{108}{2} = 54$ inches.

The perimeter is $9 + 12 + 15 = 36$ inches.

How can you be sure there are no other solutions?

Possible Discussion Questions

1. It is important for students to develop strategies for estimating length and area. What are some of the things you can do to help them develop those strategies?
2. Some educators argue that the importance of measurement in the mathematics curriculum is that it is the content that brings together the two main branches of math—number and geometry. How central a role do you think measurement should play in the curriculum?
3. Make a list of each step that you would need to use to measure the length of the edge of a table that is longer than your ruler.
   - Why might it be advisable to delay the introduction of ruler use?
   - How can you help students make the link between measuring with units and using a ruler?
4. Some teachers argue that you cannot teach about area until students know about length. Why might that not be true?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 18.
2. Complete one of the *Applying What You've Learned* questions, 9, 10, or 11 (page 527) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Study your standards and curriculum. At what grade level is each of these topics introduced?
   - Standard units for capacity or liquid volume, volume, and mass
   - Measurement of angles
   - Volume and area formulas

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Chapter 18

Volume, Mass, Time, and Angles

Chapter Summary

The main purpose of the chapter is to make connections between what is already known about measuring length and area to the study of volume, mass, time, and angles.

Key ideas:

1. The importance of all three stages of measurement instruction for volume, mass, time, and angles
2. The relationship (or lack of relationship) between various types of measurement
3. How to develop formulas meaningfully

Before the Meeting

Ask participants to do the following:

1. Read Chapter 18.
2. Complete one of the *Applying What You've Learned* questions, 9, 10, or 11 (page 527) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Study your standards and curriculum. At what grade level is each of these topics introduced?
   - Standard units for capacity or liquid volume, volume, and mass
   - Measurement of angles
   - Volume and area formulas

During the Meeting (grid paper and scissors needed)

Sharing the Pre-work

Ask participants to partner up with someone and share what they found in their standards and curriculum. Do the particular grade levels when these topics are introduced seem reasonable to you? Why or why not?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 477)

Ask participants to share what they noticed about the volume when a larger square was cut out of each corner.

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If you cut 1 square out of each corner and fold to make an open box, it will hold 32 cm^3.

If you cut squares that are 2 by 2 out of each corner and fold to make an open box, it will hold 24 cm^3.

By having a larger square cut out of the corner, the base becomes much smaller. It goes from 8 by 4 = 32 cm^2 to only 6 by 2 = 12 cm^2. The decrease in size in base area is not made up for by having 2 layers.

This problem is part of a class of problems called optimizing problems. Others are, for example, "What is the greatest area for a given perimeter?" or "What is the greatest volume for a given surface area?" Why might these problems be important for students to meet?

Possible Discussion Questions

1. Why is it important to use actual 3-D objects, and not just pictures, when teaching students about volume and mass?
   - What are some interesting activities involving 3-D objects that are appropriate for students at each of the three stages of teaching of volume and mass?
   - How can we support students in eventually transitioning from using 3-D objects to 2-D pictures?
2. Do you think it might be useful to teach volume and mass together, or do you think they should be taught separately? What are your reasons?
3. The clock is sometimes used as a "compass." For example, you might say that a building is at 2 o'clock. Do you think this use of the clock enhances or detracts from students' understanding of telling time?
   - What about using the clock to model other math concepts, such as fractions or angles? Do these uses enhance or detract from students' understanding of telling time?
4. Both time and angle measurement are based more on the Imperial system of measurement than on the metric system. What number skills are supported by work with each set of measurement systems (Imperial and metric)?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

After the Meeting

Ask participants to do the following:

1. Read Chapter 19.
2. Complete either *Applying What You've Learned* question 11 or 12 (page 584) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Visit slowrevealgraphs.com and browse the Learn About It section to learn about this instructional routine. Then browse the Resources for the Classroom section to find a graph that you think would be interesting to share with students using this routine.

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Chapter 19

Data

Chapter Summary

The main purpose of the chapter is to explore data collection, data description, data display, and data analysis. In our data driven society, ensuring future citizens understand data is important.

Key ideas:

1. How and why Venn diagrams are useful
2. Some issues that should be raised as students collect data to analyze
3. When various statistics are useful or not
4. The critical importance of visual displays in making data meaningful to a user/reader
5. How different types of graphs are more appropriate at different developmental levels and why
6. Why a focus on inference is as (if not more) important than a focus on construction of data displays

Before the Meeting

Ask participants to do the following:

1. Read Chapter 19.
2. Complete either *Applying What You've Learned* question 11 or 12 (page 584) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Visit slowrevealgraphs.com and browse the Learn About It section to learn about this instructional routine. Then browse the Resources for the Classroom section to find a graph that you think would be interesting to share with students using this routine.

During the Meeting

Sharing the Pre-work

Ask participants to partner up with someone and share the graph they chose and what they learned about the Slow Reveal Graph routine. How might you use this site/routine to help students learn about the world and, at the same time, become better interpreters of graphs?

It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

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Do Some Math Together: Solve the Chapter Problem (page 529)

What strategy did participants used to solve the chapter problem? Why was that strategy appropriate?

About 25 schools have fewer than 200 students.

The total number of schools is 100. The red bar is 2.5 units high, the green is 4.5 units high, and the yellow is 3 units high. That's a total of 10 units to represent the 100 schools, so each unit represents about 10 schools.

$10 \times 2.5 = 25$

What if the graph had shown the enrollment in 200 schools? How would that change your answer? 150 schools?

Possible Discussion Questions

1. A student in your Grade 3 class is preparing to do a survey to find out how fellow students get to school. The student's question is: How do you get to school?
   - What are some potential problems with this question?
   - How might you get the student to think of those potential problems?
   - How might you help the student consider what the nature of the sample should be?
2. Look at the four data displays on page 552.
   - What do all four data displays have in common?
   - What are the differences between the four data displays? How do they make certain ideas about this set of data more or less obvious?
   - If you wanted to share this data, what might influence your decision regarding which type of display to use?
3. How can you use work with graphing and data display to support students' number sense development?
4. How would you help students better understand when each different measure of central tendency and measure of spread is most appropriate?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

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After the Meeting

Ask participants to do the following:

1. Read Chapter 20.
2. Complete either *Applying What You've Learned* question 11 or 12 (page 605) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Reflect on the following questions and bring your written response to the meeting:
   - What are three of the most important ideas from the book that every teacher should know about?
   - What are two things you've already tried or that you want to try as a result of this book study?
   - What is one lingering question you still have?

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Chapter 20

Probability

Chapter Summary

The main purpose of the chapter is to explore the development of both experimental and basic theoretical probability concepts.

Key ideas:

1. Why the results of a random event cannot be predicted
2. Why work with experimental probability should precede work with theoretical probability
3. Why work with qualitative probability terms is valuable
4. When experimental probability makes more sense to use and when theoretical probability makes more sense to use

Before the Meeting:

Ask participants to do the following:

1. Read Chapter 20.
2. Complete either *Applying What You've Learned* question 11 or 12 (page 605) to interact with a student or a teacher (or both!) about the content of the chapter.
3. Reflect on the following questions and bring your written response to the meeting:
   - What are three of the most important ideas from the book that every teacher should know about?
   - What are two things you've already tried or that you want to try as a result of this book study?
   - What is one lingering question you still have?

During the Meeting

Sharing the Pre-work

Because this is the final meeting, it is important to bring some closure to the experience while also helping participants consider next steps in their journey as mathematics educators. Ask participants to partner up with someone and share their reflections. After participants have had a chance to share with their partner, open up discussion to the whole group to share key takeaways and lingering questions from the book study. It's not important that you have answers to all their lingering questions. The more important point is that lingering questions give participants direction about where they might go next as they continue to grow and refine their craft.

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It might be fruitful to discuss what participants learned in their interviews at the start, or you might invite participants to share these perspectives throughout the meeting.

Do Some Math Together: Solve the Chapter Problem (page 587)

Before participants attempt the problem ask: How likely do you think it is that at least one student in a class was born on the 11th day of a month? Why do you think that?

The probability is 0.633.

Imagine there are 30 students in the class. There are 12 days in the year that are the 11th day of a month, and there are 353 days (in a non-leap year) that are not. For each of the 30 students, the probability of not being born on the 11th is $\frac{353}{365} = 0.967$. To get the probability that none of the 30 students was born on the 11th, you multiply 0.967 by itself 30 times. That would be 0.367.

That means the probability that at least one person was born on the 11th is $1 - 0.367 = 0.633$.

How did your initial prediction compare to the probability we calculated? Based on your experience today, what is one thing you might consider next time you make a prediction involving probability?

Possible Discussion Questions

1. What do you see as the most important probability ideas for students to learn?
2. Some people argue that probability is too abstract for young students to deal with. Argue both sides of the debate. Where do you stand?
3. Look at the list of probability vocabulary on page 604.
   - Which terms do students need to be familiar with at your grade level?
   - Are there any important terms missing from this list that are important at your grade level?
   - How might you ensure that students are comfortable with these terms?
4. At some point, we introduce the notion of recording a probability as a fraction. How might a teacher integrate the exploration of fraction concepts with an exploration of probability concepts?
5. Discuss as many of the questions from the *Applying What You've Learned* section as you find appropriate, depending on your circumstances.

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