cards. They can also be used for a quick game of "Which Is More?" or "Which Is Less?" (Some people call the game "War," and we also call it "Top-It," a term from *Everyday Mathematics* [University of Chicago School Mathematics Project 2007].) They use the dot cards and ten-frames for identifying amounts, but also for developing part-whole relationship understandings. For example, in addition to asking, "How many?" they also ask, "How many are missing?" or, "How many more to make a five?" or, "How many more do you need to make a ten?"

Mary Anne and Christy use these Math Box tools with small groups, in addition to other whole-class mathematics routines, as a way to differentiate the needs of their learners. Using routines in this way helps them to assess their students continuously and to individualize instruction on a day-to-day basis. It can be overwhelming to differentiate lessons for a wide range of learners, but routines are one small component to your math workshop. If differentiation for your entire math block feels daunting, differentiating for your number sense routines is one way to start. And, using routines in this way is a very responsive way to plan your instruction based on students' strengths and needs.

ORGANIC NUMBER LINE

The Organic Number Line routine helps second- and third-grade students develop a mental linear model for fractions and decimals. (See Box 4.7 for definitions of models used in fractions work.) The Organic Number Line is one section of a "whole number" number line; with it, you magnify the number line from 0 to 2 (see Figure 4.8). It is "organic" because your students

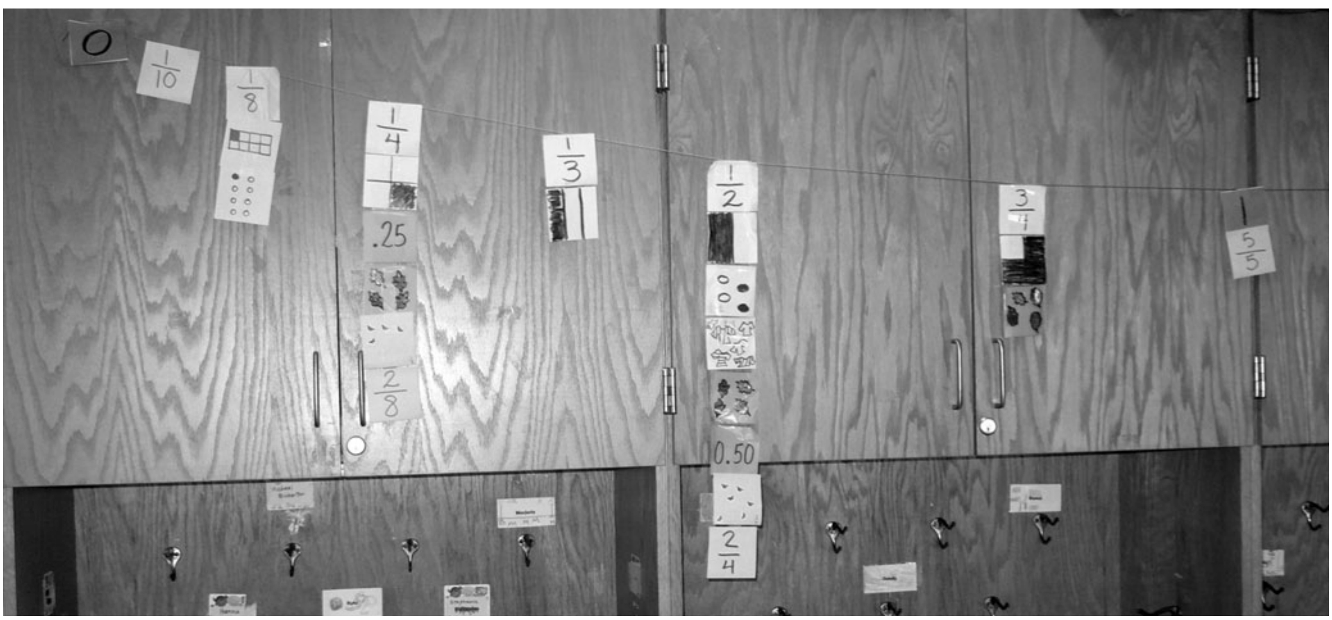


Figure 4.8
An Organic Number Line from My Third-Grade Class

continuously add to it throughout the year. It is ever changing based on the experiences in your class.

To create the Organic Number Line in my third-grade class, we used a string that was six feet long and cards labeled with numerals and pictures. I introduced the number line by placing the card with the numeral 0 on one end and asking a student to put the card with the numeral 1 anywhere on the number line. Lizbeth volunteered, and placed the 1 fairly close to the 0.

"If Lizbeth put 1 there, where does the $\frac{1}{2}$ go and how do you know it goes there?" I asked. Adib took the $\frac{1}{2}$ card, placed it halfway between the 0 and the 1, and explained that he estimated that this point was about halfway between 0 and 1. Next, I moved the 1 and asked, "Where does the $\frac{1}{2}$ go now?" The children could answer the question by identifying the new point, and could explain how they knew where $\frac{1}{2}$ went.

"So, why do those points move?" I asked. This question spurred a discussion about the concept of the whole. The children talked about how the whole changes depending on where the 0 and 1 are located. I explained, "You identified the $\frac{1}{2}$ easily no matter where I put the 0 and 1. That means that the $\frac{1}{2}$ is your benchmark. You can find out where other fractions lie on the number line by using that benchmark."

During this initial discussion, other children added $\frac{1}{4}$, $\frac{1}{8}$, $\frac{3}{4}$, $\frac{1}{3}$, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{15}{16}$, and $\frac{1}{100}$ using the benchmark $\frac{1}{2}$ or using their knowledge about the relationships among halves, fourths, and eighths. They were able to add these points to the Organic Number Line because they had had many experiences with these fractional amounts through problem solving before we began this routine. They had solved many story problems about sharing pizzas, brownies, and pies ("fair share"), made fraction bars out of paper (a lesson from Marilyn Burns [2001]), and played with set models, such as, "What fraction of this group of students is boys?" They had had concrete experiences with many fractional amounts and had developed visual models of these amounts in their heads. Now they were ordering these symbolic numbers on a number line based on their concrete and visual experiences with fractions.

Students require strong visual understandings of fractions prior to embarking on the Organic Number Line routine. Without a mental image of what $\frac{1}{2}$ or $\frac{3}{4}$ looks like, an understanding of what $\frac{1}{4}$ of a pie means, or an understanding of the idea that fraction and decimal numbers are part of a whole amount, students will struggle with understanding this linear model. Children need a variety and plethora of experiences with "fair share" story problems. They need visual understandings of region models of fractional amounts (like pies, brownies, and pizzas) and set models of fractional amounts (like the fraction of crayons that are blue) as well as clear mental models of important benchmarks, like halves, thirds, and fourths.

In our third-grade classroom, we had a special place for the Organic Number Line, where it was displayed and evolved throughout the year. We

BOX 4.7

Definitions of Visual Models for Work with Fractions

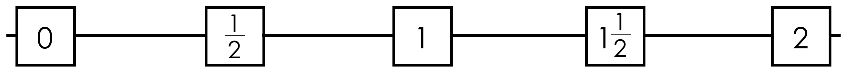
Linear model: A number line or ruler model on which fractions are often represented by fraction bars or fraction strips or on the ruler.

Region model (or area model): Whole amounts such as pies, pizzas, and cookies, often represented by pattern blocks or geoboards, on which fractions are represented as segments of the whole. (For example, "This is one-quarter of the pie.")

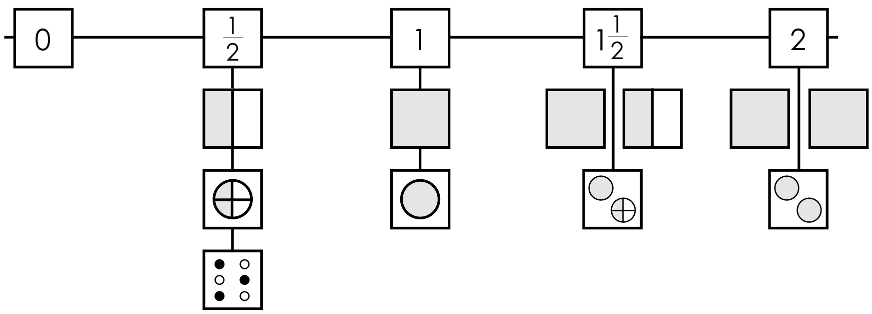
Set model: Individual, distinct objects (e.g., counters, chips, toys, or people) that are grouped together to demonstrate a fractional amount. (For example, "Three out of four people are wearing red.")

usually sat in a circle during our whole-class routines to better facilitate a class discussion; however, we had special meetings for our Organic Number Line routine and gathered in a "clump" (group) in front of it. Sometimes I led the discussion. In these cases, I held up a card that showed either a picture of a fractional amount or a written numeral representing an amount, and I asked the students where they thought the card belonged on the Organic Number Line. Other times, various students (sometimes the whole class if we were doing a longer warm-up) had cards and decided individually or as a whole group where their cards belonged and why.

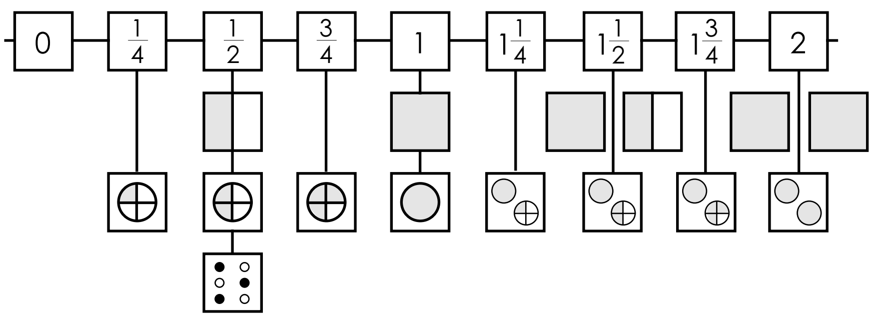
It is best to start your discussions of the Organic Number Line with the half and whole benchmarks:



Then, think of ways that halves and wholes are represented, and add to your number line:



Later, add the quarter benchmarks:



After these initial benchmarks are established, hold discussions on subsequent days about where $\frac{1}{3}$ is located on the number line. Ask students, *How did you figure that out?* and *How was your thinking different from finding the $\frac{1}{2}$ point?* Discuss how they know where $\frac{1}{8}$ and $\frac{1}{16}$ are located and how their strategies for finding these points changed.

Interesting discussions take place after this initial introduction. In this particular routine, guide your class to set up the structure of the number line using common benchmarks, then plan which cards you will use to highlight specific mathematical ideas. Here are some examples of big ideas in fractions that you can teach and questions you can discuss using the Organic Number Line:

Big Idea: Benchmark and Counting Sequences

- *Where does this number go on our number line? How do you know?*
- *What numbers can you think of that go between $\frac{1}{2}$ and 1? How do you know that number goes between these two benchmarks?*

Big Idea: Equivalency

- *You said that $\frac{2}{4}$ goes here with $\frac{1}{2}$. Prove that $\frac{2}{4}$ and $\frac{1}{2}$ are equivalent.*

Big Idea: The Whole and Parts of a Whole

- *Why does $\frac{1}{2}$ go here and $1\frac{1}{2}$ go over here?*
- *Are this half and this half the same amount?* (Show two models representing $\frac{1}{2}$, each with a different whole.) *Prove it!*

Big Idea: Linear Model as a Tool

- *What on the number line helped you figure that out?*

Big Idea: Region and Set Models

- *Here we represent $\frac{1}{2}$ with this picture:*

Can you think of another way to show that number? (Students might show $\frac{1}{2}$ as a set model by showing 4 yellow leaves out of 8 leaves total or show $\frac{1}{2}$ as a region model represented with a pizza cut into sixteenths, with 8 pieces gone.)

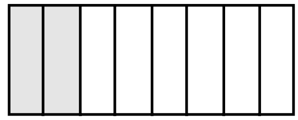
Big Idea: Doubling and Halving

- *What is half of that amount? Where does that fraction go on the number line?*
- *How did you know that $\frac{2}{4}$ goes in the same spot at $\frac{1}{2}$?*
- *How did you know that $\frac{1}{8}$ should be halfway between $\frac{1}{4}$ and 0?*

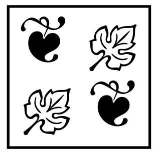
As I plan for discussions around these big ideas, I also think carefully about which cards to use—both the amount and the representation (symbolic or pictorial). If I know that my students have a visual model in their heads for halves, fourths, eighths, and thirds, I'll use a symbolic notation:

$\frac{1}{2}$ $\frac{3}{4}$ $\frac{4}{8}$ $\frac{1}{3}$

If some students are struggling with knowing where $\frac{2}{8}$ goes, I'll use a picture of a region model (here, showing $\frac{2}{8}$ of a rectangular pan of brownies):



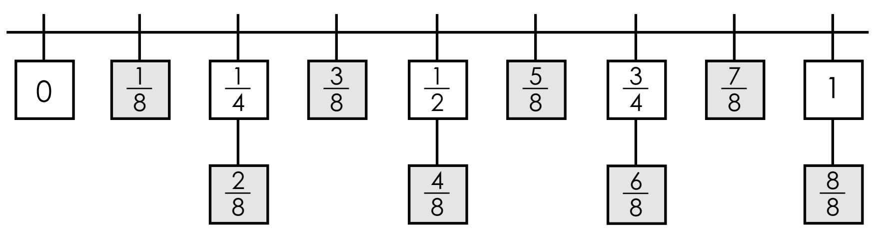
I use stickers to make cards that show set models (here, two of the four leaves are yellow, which is equivalent to a half, so this card goes at the point on the number line that represents $\frac{1}{2}$).



Twp out of four leaves are yellow.
Half of the leaves are yellow.

Planning these routines based on students' concrete and visual experiences and knowledge will help them better understand fractions as a linear model—fractions on a number line. The Organic Number Line is another excellent model (in addition to visual region/area and set models) to help students develop a fuller understanding of fractional ideas. Some of the benefits of using this routine include the following:

- *Stronger linear and measurement understandings of fractions*. In the past, I often observed that my students could recognize and compare fractions like $\frac{1}{4}$ and $\frac{1}{8}$ and discuss the idea that $\frac{1}{8}$ is less than $\frac{1}{4}$, but they struggled with using fractions when measuring. For example, when they used a ruler or counted by a fractional amount, they often were stumped by $\frac{1}{8}$ being smaller than $\frac{1}{4}$. After working with the Organic Number Line, they were better able to apply what they knew about $\frac{1}{8}$ and $\frac{1}{4}$ because they understood both what that quantity looked like (as a region model) and where it fit in a sequence in a linear model. The Organic Number Line helped them make connections between their visual understanding of fractions and the linear, measurement quality of fractions.
- *Increased skill and flexibility when counting with fractions*. Students often referred to the Organic Number Line when we counted by fractional amounts during Count Around the Circle. The model helped them see the "jumps" as they counted, such as when we counted by eighths:



- *Deeper and broader understanding of equivalent fractions*. Counting by fractional amounts, like eighths, combined with the linear model of the number line as a visual, helped my students develop an understanding of equivalent fractions. They were better able to recognize that $\frac{1}{2}$ is the same as $\frac{4}{8}$, $\frac{2}{4}$, and $\frac{5}{10}$ and that all fractions have multiple names or are represented in multiple ways.
- *Increased ability in estimation with fractions and better understanding of relationships among fractions*. Students were more apt to recognize that $\frac{1}{8}$ is closer to 0 than it is to 1 and that $\frac{7}{8}$ is closer to 1 than it is to 0. Students referred to the Organic Number Line to prove their estimates, citing the distance between fractions and the relationships among fractions.
- *Increased ability to use benchmarks to compare fractional amounts*. Once we started using the Organic Number Line, students were more easily able to use benchmarks like $\frac{1}{2}$ and $\frac{1}{4}$ to compare fractions without the support of a visual.
- *More exact measurements in math and science*. While using the Organic Number Line, I noticed that my third graders were using fractions to be more exact in their measurements during science. For example, we measured winter wheat over several months, and as we worked with the Organic Number Line during our number sense routines, their abilities to use exact measurements improved greatly. They were applying their sense of fractional amounts and their understanding of the linear model to a variety of measurement problems during science discussions and in their observational drawings and notes (see Figure 4.9).

Early foundations in understanding and visualizing fractional amounts are essential for developing complex ideas later. Children need to be able to understand part-whole relationships, relationships among fractions, relationships among fractions and decimals, comparisons of fractions to the whole amount, and equivalency. The linear model especially supports children in seeing the patterns in counting by fractions, which helps them gain more solid understandings of the relationships among fractions and relationships between a rational number (fraction/decimal) and a whole number. The Organic Number Line is one more component in helping students develop these concepts throughout the year. Use Count Around the Circle or Choral Counting while using the Organic Number Line as a reference, then facilitate discussions about the positions of numbers and their relationships on the Organic Number Line. Keep it organic and keep it growing.

• • •

Counting routines help students learn sequences and understand where numbers fall in the number line and in relation to one another. Students not only learn how to count but also build understandings of how the base ten, place-value number system works. They gain insight into relationships

Figure 4.9
Marjorie measures winter wheat using fractions.