Building Fact Fluency

A TOOLKIT FOR MULTIPLICATION & DIVISION

RESEARCH BASE OVERVIEW

Grounded in current research, *Building Fact Fluency: A Toolkit for Multiplication & Division* is designed and sequenced so that students will accumulate number facts while developing deep understanding of the operations and their fundamental properties. Here's a quick look at just some of the 50+ works by noted researchers, educators, and mathematics organizations that support the philosophy and lesson structure of *Building Fact Fluency.*

1. *Building Fact Fluency* supports conceptual understanding—not just memorization.

As students build from what they know to figure out what they don't yet know, they learn new math facts and develop an understanding of the operations, making connections and noticing relationships along the way.

RESEARCH SUPPORTS

"The development of procedural fluency, including the learning of basic number combinations and formulas, must begin with and build from a solid foundation of deep conceptual knowledge. This teaching approach is not only essential for developing positive mathematical identities and strong agency but is a necessary prerequisite. The tendency to rush children to prematurely use procedures or memorize facts and formulas without sufficient understanding robs children of opportunities to grow confidence in themselves as mathematical knowers and doers. Procedural fluency develops gradually over many years and must always keep meaning and sense making at the forefront."

National Council of Teachers of Mathematics. 2020. *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations.* Reston, VA: National Council of Teachers of Mathematics.

"Being fluent means that students are able to choose flexibly among methods and strategies to solve contextual and mathematical problems, they understand and are able to explain their approaches, and they are able to produce accurate answers efficiently. Fluency builds from initial exploration and discussion of number concepts to using informal reasoning strategies based on meanings and properties of the operations to the eventual use of general methods as tools in solving problems."

National Council of Teachers of Mathematics. 2014. *Principles to Actions: Ensuring Mathematical Success for All*. Reston, VA: National Council of Teachers of Mathematics.

"Psychologists have long known that people more easily learn a body of knowledge by focusing on its structure (i.e., underlying patterns and relationships) than by memorizing individual facts by rote. Furthermore, psychologists have long known that well-connected factual knowledge is easier to retain in memory and to transfer to learning other new but related facts than are isolated facts."

Baroody, Arthur J. 2006. "Why Children Have Difficulties Mastering the Basic Number Combinations and How to Help Them." *Teaching Children Mathematics* 13 (1): 22–31.

“Children who struggle to commit basic facts to memory often believe that there are ‘hundreds’ to be memorized because they have little or no understanding of the relationships among them. Children who commit the facts to memory easily are able to do so because they have constructed relationships among them and use these relationships as shortcuts.”

Fosnot, Catherine Twomey, and Maarten Dolk. 2001. *Young Mathematicians at Work: Constructing Multiplication and Division.* Portsmouth, NH: Heinemann.

“As students become better able to identify and make sense of multiplicative situations, they tend to start applying more sophisticated solution strategies. The progression of solution strategies from direct counting to repeated addition/subtraction to multiplicative operations is neither linear nor clear-cut. The strategies students employ depend on the size of the numbers in the problem, the semantic structure or subcategory of the problem, and the quantity that is unknown. Furthermore, the progression from additive strategies (counting and repeated addition/subtraction) to the use of multiplicative strategies requires several cognitive shifts in students’ thinking.”

Chapin, Suzanne H., and Art Johnson. 2006. *Math Matters: Understanding the Math You Teach, Grades K–8*. 2nd ed. Sausalito, CA: Math Solutions.

“The issue here is not whether facts should eventually be memorized but how this memorization is achieved: by drill, practice, and memorization, or by focusing on relationships?”

Fosnot, Catherine Twomey, and Maarten Dolk. 2001. *Young Mathematicians at Work: Constructing Multiplication and Division.* Portsmouth, NH: Heinemann.

“In the elementary grades, the study of arithmetic is central to the curriculum. Deep understanding of this territory rests on three pillars: understanding numbers, developing computational fluency, and examining the behavior of the operations.”

Russell, Susan Jo, Deborah Schifter, and Virginia Bastable. 2011. *Connecting Arithmetic to Algebra: Strategies for Building Algebraic Thinking in the Elementary Grades.* Portsmouth, NH: Heinemann.

“Children learn number facts by noticing relations among number facts.”

Carpenter, Thomas P., Elizabeth Fennema, Megan Loef Franke, Linda Levi, and Susan B. Empson. 2015. *Children’s Mathematics: Cognitively Guided Instruction*. 2nd ed. Portsmouth, NH: Heinemann.

“Students make use of *known facts* and relationships to *derive unknown facts*.”

Van de Walle, John A., Karen S. Karp, and Jennifer M. Bay-Williams. 2019. *Elementary and Middle School Mathematics: Teaching Developmentally.* 10th ed. London: Pearson.

“One of the key components of understanding is being able to explain why a procedure works or why a particular statement is true.”

Carpenter, Thomas P., Megan Loef Franke, and Linda Levi. 2003. *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School.* 2nd ed. Portsmouth, NH: Heinemann.

2. *Building Fact Fluency: A Toolkit for Multiplication & Division* provides multiple opportunities for students to explore math concepts in different contexts and build connections across ideas. Students relate ideas and build connections as they learn number combinations in and out of contexts, with and without tools, through problem-solving and algebraic reasoning, in abstract practice and metacognitive reflection, and within eleven different routines.

RESEARCH SUPPORTS

"When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding and enhanced problem-solving abilities."

National Council of Teachers of Mathematics. 2014. *Principles to Actions: Ensuring Mathematical Success for All.* Reston, VA: National Council of Teachers of Mathematics.

"Our job as teachers is to help students connect procedures, properties of operations, and understanding of place value rather than have them learn these concepts as separate, compartmentalized pieces of knowledge."

Russell, Susan Jo. 2000. "Developing Computational Fluency with Whole Numbers in the Elementary Grades." *Teaching Children Mathematics* 7 (3): 154–158.

"If children only needed to solve written multiplication problems already set up for them, they would be able to learn to multiply without paying too much attention to multiplicative thinking. However, that is not what will make knowledge of multiplication facts useful. Children need to understand multiplication situations and the language associated with them."

Richardson, Kathy. 2012. *How Children Learn Number Concepts: A Guide to the Critical Learning Phases.* Bellingham, WA: Math Perspectives.

"The different meanings of multiplication and division can only be distinguished through appropriate contexts."

Small, Marian. 2019. *Understanding the Math We Teach and How to Teach It, K–8.* Portsmouth, NH: Stenhouse.

"Generalizing across problems, across models, and across operations is at the heart of models that are tools for thinking."

Fosnot, Catherine Twomey, and Maarten Dolk. 2001. *Young Mathematicians at Work: Constructing Multiplication and Division.* Portsmouth, NH: Heinemann.

"The artificial separation of arithmetic and algebra deprives students of powerful ways of thinking about mathematics in the early grades and makes it more difficult for them to learn algebra in the later grades."

Carpenter, Thomas P., Megan Loef Franke, and Linda Levi. 2003. *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School.* 2nd ed. Portsmouth, NH: Heinemann.

"The contexts of tasks allow all students entry into the situation and allow all students to problematize the situations."

Hiebert, James, Thomas P. Carpenter, Elizabeth Fennema, Karen C. Fuson, Dine Wearne, Hanlie Murray, Alwyn Olivier, and Piet Human. 1997. *Making Sense: Teaching and Learning Mathematics with Understanding.* Portsmouth, NH: Heinemann.

3. Lesson strings in the *Building Fact Fluency* toolkit provide interleaved, varied, and spaced activities and tasks to develop students' ability to learn, apply, and practice different strategies over time. The intentional variety of tasks and thoughtful sequencing are designed to build understanding, maximize retention, and support transfer to future math learning.

RESEARCH SUPPORTS

"The daily practice—the everyday, connected interactions—with numbers, amounts, patterns, and relationships will build your students' number sense and develop their mathematical understanding."

Shumway, Jessica. 2018. *Number Sense Routines: Building Mathematical Understanding Every Day in Grades 3–5.* Portland, ME: Stenhouse.

"Students who learn their basic facts through varied practice learn not only the facts but also a way of thinking and working in mathematics that is very useful beyond the context of learning basic facts."

Crespo, Sandra, Andreas O. Kyriakides, and Shelly McGee. 2005. "Nothing 'Basic' About Basic Facts: Exploring Addition Facts with Fourth Graders." *Teaching Children Mathematics* 12 (2): 61–67.

"Practice should not be 'meaningless drill' but should occur in a context of making sense of the situation and the number relationships."

Clements, Douglas H., and Julie Sarama. 2014. *Learning and Teaching Early Math: The Learning Trajectories Approach.* 2nd ed. New York: Routledge.

"While practicing is vital to learning and memory, studies have shown that practice is far more effective when it's broken into separate periods of training that are spaced out. The rapid gains produced by massed practice are often evident, but the rapid forgetting that follows is not. Practice that's spaced out, interleaved with other learning, and varied produces better mastery, longer retention, and more versatility."

Brown, Peter C., Henry L. Roediger III, and Mark A. McDaniel. 2014. *Make It Stick: The Science of Successful Learning*. Cambridge, MA: Belknap Press.

"The research shows unequivocally that mastery and long-term retention are much better if you interleave practice than if you mass it."

Brown, Peter C., Henry L. Roediger III, and Mark A. McDaniel. 2014. *Make It Stick: The Science of Successful Learning*. Cambridge, MA: Belknap Press.

"The basic idea is that *varied* practice…improves your ability to transfer learning from one situation and apply it successfully to another."

Brown, Peter C., Henry L. Roediger III, and Mark A. McDaniel. 2014. *Make It Stick: The Science of Successful Learning*. Cambridge, MA: Belknap Press.

"The increased effort required to retrieve the learning after a little forgetting has the effect of retriggering consolidation, further strengthening memory."

Brown, Peter C., Henry L. Roediger III, and Mark A. McDaniel. 2014. *Make It Stick: The Science of Successful Learning*. Cambridge, MA: Belknap Press.

4. *Building Fact Fluency* is designed to disrupt and replace inequitable practices and support shifts toward more equitable instruction. Traditional fact fluency instruction has served as a gatekeeper—many students are not offered access to rich, challenging mathematics unless and until they have memorized their facts, and these students are often disproportionately from historically excluded communities. *Building Fact Fluency* is intended to disrupt this structure so that all students engage in rich, challenging, problem-based mathematics right from the outset.

RESEARCH SUPPORTS

"Schools and districts must prioritize meaningful learning of basic number combinations and remove inequitable structures and practices (e.g., timed tests, drills, rote memorization) that have unintended and lifelong negative consequences on children. This also demands an understanding of how fluency is developed and positioning the learning of basic number combinations within a broader context of number and operation sense."

National Council of Teachers of Mathematics, 2020. *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations.* Reston, VA: National Council of Teachers of Mathematics.

"Each and every child should develop deep mathematical understanding as confident and capable learners; understand and critique the world through mathematics; and experience the wonder, joy, and beauty of mathematics... Early childhood and elementary mathematics should dismantle inequitable structures, including ability grouping and tracking, and challenge spaces of marginality and privilege... Mathematics instruction should be consistent with research-informed and equitable teaching practices that nurture children's positive mathematical identities and strong sense of agency...Early childhood settings and elementary schools should build a strong foundation of deep mathematical understanding, emphasize reasoning and sense making, and ensure the highest-quality mathematics education for each and every child."

National Council of Teachers of Mathematics, 2020. *Catalyzing Change in Early Childhood and Elementary Mathematics: Initiating Critical Conversations.* Reston, VA: National Council of Teachers of Mathematics.

"By 'offering an increased number of pathways and points of entry' to students, we can broaden their conception of what it means to be good at math and position all our students as competent mathematicians."

Aguirre, Julia Maria, Karen Mayfield-Ingram, and Danny Bernard Martin. 2013. *The Impact of Identity in K–8 Mathematics: Rethinking Equity-Based Practices.* Reston, VA: National Council of Teachers of Mathematics.

"Developing mathematical thinking in the elementary grades puts students on a path to learning mathematics with understanding so that algebra is a gateway to opportunity, not a gate that blocks their way."

Carpenter, Thomas P., Megan Loef Franke, and Linda Levi. 2003. *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School.* 2nd ed. Portsmouth, NH: Heinemann.

5. *Building Fact Fluency* is built on best practices in the teaching and learning of mathematics. Student discourse, representations, sense making, argumentation, and the rest of the process and practice standards are the center of effective mathematics classrooms. The same can and should be true during fluency work.

RESEARCH SUPPORTS

"Instead of acting as the main source of mathematical information and the evaluator of correctness, the teacher now has the role of selecting and posing appropriate sequences of problems as opportunities for learning, sharing information when it is essential for tackling problems, and facilitating the establishment of a classroom culture in which pupils work on novel problems individually and interactively, and discuss and reflect on their answers and methods. The teacher relies on the reflective and conversational problem-solving activities of the students to drive their learning."

Hiebert, James, Thomas P. Carpenter, Elizabeth Fennema, Karen C. Fuson, Dine Wearne, Hanlie Murray, Alwyn Olivier, and Piet Human. 1997. *Making Sense: Teaching and Learning Mathematics with Understanding.* Portsmouth, NH: Heinemann.

"Something wonderful happens when students learn they can make sense of mathematics in their own ways, make mathematically convincing arguments, and critique and build on the ideas of their peers."

Humphreys, Cathy, and Ruth Parker. 2015. *Making Number Talks Matter: Developing Mathematical Practices and Deepening Understanding, Grades 3–10.* Portland, ME: Stenhouse.

"When children look for sameness and difference; when they work hard to put their ideas into words; when they evaluate whether somebody else's justification makes sense; when they wonder;…children engage in real mathematical thinking."

Danielson, Christopher. 2016. *Which One Doesn't Belong? A Shapes Book and Teacher's Guide.* Portland, ME: Stenhouse.

"There are standards in mathematics for deciding whether a mathematical statement is true or not. An important goal of mathematics instruction is to help students address how disputes in mathematics are resolved and understand what is required to show that a mathematical statement is true."

Carpenter, Thomas P., Megan Loef Franke, and Linda Levi. 2003. *Thinking Mathematically: Integrating Arithmetic & Algebra in Elementary School.* 2nd ed. Portsmouth, NH: Heinemann.

"Speed with arithmetic skills has little, if anything, to do with mathematical power. The more important measure of children's mathematical prowess is their ability to use numbers to solve problems, confidently analyze situations that call for the use of numerical calculations, and arrive at the reasonable numerical decisions that they can explain and justify."

Burns, Marilyn. 1995. "In My Opinion: Timed Tests." *Teaching Children Mathematics* 1 (7): 408–409.

"We have the research evidence that shows students can learn math facts much more powerfully with engaging activities; now is the time to use this evidence and liberate students from mathematics fear."

Boaler, Jo. 2016. *Mathematical Mindsets: Unleashing Students' potential Through Creative Math, Inspiring Messages, and Innovative Teaching.* San Francisco, CA: Jossey-Bass.