CHAPTER 4

Children Are Brilliant Mathematicians

One evening when my son, Griffin, was six years old, he was eager to have me do something with him (we have both since forgotten what that thing was). I was busy and told him I needed two more minutes to finish up. In order to hold me accountable, he proclaimed that he would count to 120 and that he would run laps of our house's first floor while doing so. Perhaps this was mainly a clever gambit for gaining my attention, and if it was, I bit hard by getting out my Flip camera (this was 2010) and filming him in action. You can watch the resulting video at https://vimeo.com/23501648. His younger sister, Tabitha—four years old at the time—wanted in on the action. She insisted I make a video of her counting, too (https://vimeo.com/23543507). The six-year-old ran and counted nearly flawlessly; the four-year-old made a number of predictable counting errors along the way as she counted (almost) to thirty.

For many children, counting is the introduction to mathematics. Sometimes adults see counting as a prerequisite for mathematical activity—a thing that is straightforward and unproblematic, and that children must master before they can do real math. In fact, counting is more complicated and interesting than this. Counting *is* real mathematics, and it is not complete when the child can correctly count collections of individual objects.

I used to think that the surface features of these videos told a fairly complete story of children's counting development. Griffin had mastered the counting sequence to 120 (and probably far beyond); Tabitha struggled in the teens, skipped twenty, and went straight to twenty-one, and so forth. As I have watched these children develop, and as I have studied children's learning more broadly, I now see much more going on in this scene. Now I see Griffin counting seconds while Tabitha is reciting number words. I notice that seconds are rather abstract units to count, and so they afford Griffin and me fewer opportunities to play with grouping and naming than if he were counting something more tangible such as eggs, shoes, or pizza slices.

This chapter aims to provide the reader with similar shifts in perspective. I hope to help you uncover complexity in what seems simple on the surface; to find wonder in things you thought you already knew. In service of this goal, I'll invite you into classrooms and conversations where children are doing rich mathematical thinking as they clarify the things they are counting, use the structures they notice, and uncover deep, important mathematical relationships.

WHAT CAN WE LEARN FROM SHOELACES?

While visiting a local school to talk about *How Many?* with students of a variety of ages, I found myself taking a straw poll of second graders. I had introduced the book to the approximately fifty seven- and eight-year-olds assembled that morning by explaining that I was interested in what *they* saw in these images and that we would be diligent and precise in our conversations. They put me to the test almost immediately when the second child I called on said that she saw four laces in Figure 4.1.

"Four ends on the laces?" I asked.

"No. Four laces," she replied.

I paused to think.

"So each shoe has two laces?" I asked.

FIGURE 4.1
Does each shoe have one lace or two?

"Yup," she said with confidence.

In moments such as this, I have learned that when one child says something unexpected it often represents the unspoken thinking of many children. Hence, the straw poll.

"Second graders! I need to know what you think here. Does each shoe have two laces, so there are four laces in this picture? Or does each shoe have one lace, so there are two total laces in this picture?" We did a show of hands, and there was a clear majority—about two-to-one—in favor of two laces per shoe.

Do not discount these results until you have polled a group of young children yourself, as I have reproduced this experiment in a wide variety of kindergarten, first-, and second-grade classrooms. Almost every time, a majority of the children vote in favor of two laces per shoe, and almost every time their teachers take this in with the same look of surprise on their faces that I must have had the first time I watched all those second-grade hands go up.

Where does this idea come from, and why is it so pervasive? A premise of *How Many?* is that children's mathematical ideas—in particular, their ideas about numbers—are based first and foremost in the experiences of their lives. By the age of seven—when many students are in second grade—most children have learned to tie their shoes. As they learn and practice shoe tying, their experience is of having two things on each shoe. One crosses over the other, then tucks underneath, et cetera. Furthermore, children at this age are growing fairly quickly. Likely, they outgrow their shoes before they need to replace a shoelace. Children have experienced the twoness of shoelaces, and they have had no opportunity to encounter their oneness. So, of course, they tend to believe that each shoe has two laces.

An alternative hypothesis exists, though. Maybe this is a question of language. Maybe when I ask, "Does each shoe have two laces?" children hear, "Does each shoe have two *ends* to its shoelace?" I have two pieces of evidence to refute this hypothesis. The first is that children themselves tell me when I ask that they do not mean two *ends*; they mean two *laces*. The second piece of evidence is that several times I have unlaced my own shoe in front of a class of primary-grade children to great delight and surprise all around when it turns out to be just one. (I have also spotted more than a few unlaced shoes following a spirited *How Many?* session.)

When I have asked this same question of fifth graders, I have found myself on the wrong end of a classroom full of the kind of icy stares older children reserve for condescending adults asking trick questions. Fifth graders—as a group—are well aware that each shoe has a single lace. It must be that sometime between the ages of seven and eleven, most children either break a shoelace or want laces that are differently colored from the ones that came in a pair of shoes.

This story illustrates that counting—a seemingly simple practice that serves in part as a gateway to mathematical activity—is grounded in children's experiences. Those second graders I told you about were not taking wild guesses about the number of shoelaces in each shoe. Those fifth graders giving me icy stares had not experienced a unit of study on the Mathematics of Clothing with a standard about correctly enumerating the laces in shoes. In both cases the ideas grew from children's experiences, and these experiences inform children's mathematical thinking in many ways and at a variety of levels.

A PAIR OF EGGS

People sometimes describe math as "a universal language." This is only partly true. In one sense, 2 + 2 = 4 expresses an idea that is universally true. But our understanding of this truth is dependent on language. How do you *know* that 2 + 2 = 4? Most likely, you turn to an example in a context. *If I have two cookies, and you give me two more cookies, then I have four cookies.* Our ways of knowing depend on language that is not universal (here *cookie* is an English word, and a concept that not all cultures share), and our ability to communicate our mathematical ideas is similarly dependent on language. Furthermore, children learning mathematics behave a lot like children learning language, but this is not because math is a universal language. It is because mathematical ideas are tightly tied to the natural language in which we express them.

For example, a routine conversation I have with children when we discuss the shoes page is about two ways of counting the shoes. Some people look at the shoes and see *two*: two shoes. Other people look at the shoes and see *one*: one pair of shoes. I emphasize that these are two ways of viewing the same thing and that part of our work together will be to notice when that's possible. One day, this discussion with a class of

1 pair of shoes

1 box

1 12-sided polygon (a dodecagon, shaped like a plus sign)

2 holes in the box's top

2 semicircles cut from the ends of the box

2 shoelaces

2 shoes

2 flaps on the box top

2 different lacing schemes (the left-footed shoe has the ends of the shoelaces going from top to bottom of the top set of eyelets; the right-footed shoe has them going from bottom to top)

3 lines of text on the box top

4 aglets

12 sides on the dodecagon

20 eyelets

About 40 yellow stitches on each shoe (of which about 20 are visible on each shoe)

About 50 small holes visible in the insoles of the two shoes combined

0 shoes

0 shoelaces

0 eyelets

0 aglets

0 elephants (although to be fair, nearly every page of this book has 0 elephants)

1 box

2 footprints

2 bold black bars in the footprints