Building Fact Fluency

A TOOLKIT FOR ADDITION & SUBTRACTION

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CORRELATION TO

Texas Essential Knowledge and Skills

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Texas Essential Knowledge and Skills
Mathematical Process Standards
– GRADES K–12 –

The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	
Apply mathematics to problems arising in everyday life, society, and the workplace.	The *Building Fact Fluency* toolkit is based on everyday contexts, e.g., crayons, shells, marbles, buttons, markers, bears, toy cars, lemonade, peppers, pizza, apples, blocks, coins, eggs, chopsticks, and tennis balls. Students solve contextualized problems in these routines: · Image Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks
Use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution.	*Building Fact Fluency* includes hundreds of problem-based lessons, which invite students to engage in all aspects of the problem-solving process. The following routines, in particular, are problem-based: · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks
Select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems.	Students have ample opportunities to select appropriate tools, techniques, and strategies to solve problems, particularly in problem-based lesson discussions, student representations, and the following routines: · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Games
Communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate.	Mathematical communication is embedded throughout *Building Fact Fluency,* especially in problem-based lesson discussions, student representations, and the following routines: · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Games

Create and use representations to organize, record, and communicate mathematical ideas.	Student representations are embedded throughout *Building Fact Fluency,* especially in problem-based lesson discussions and the following routines: · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Reflection and Journaling Teachers model a variety of representations in these warm-up routines: · Image Talks · Tool Talks · Number Talks
Analyze mathematical relationships to connect and communicate mathematical ideas.	There are many opportunities to analyze relationships, seek patterns, and make generalizations throughout *Building Fact Fluency,* especially in these routines: · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Games
Display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.	The *Building Fact Fluency* toolkit invites discussions, explanations, and justifications in both oral and written communication in every lesson, especially in these routines: · Image Talks (oral arguments with teacher modeling written language) · Tool Talks (oral arguments with teacher modeling written language) · Number Talks (oral arguments with teacher modeling written language) · Contextualized Practice Problems (students represent, write, and discuss solutions) · Anchor Problems (students represent, write, and discuss solutions) · 3-Act Math Tasks (students represent, write, and discuss solutions) · Games (oral discussion and justification) · Interviews, Conferences, Journaling and Reflection (both oral and written)

Texas Essential Knowledge and Skills Procedural Fluency Statement

– GRADES K–5 –

For students to become fluent in mathematics, students must develop a robust sense of number. The National Research Council's report, "Adding It Up," defines procedural fluency as "skill in carrying out procedures flexibly, accurately, efficiently, and appropriately." As students develop procedural fluency, they must also realize that true problem solving may take time, effort, and perseverance.	The *Building Fact Fluency* toolkit is designed to build robust number sense and fluency consistent with the "Adding It Up" definition, as described at length in the first chapter of the Facilitator's Guide. Through the *Building Fact Fluency* routines, students have thousands of opportunities to build conceptual understanding of the operations and connect mathematical ideas while they develop procedural fluency of their single-digit facts. Students with deep fact fluency have the flexibility, accuracy, and efficiency to marshal those facts appropriately in the midst of more complex problems and procedures later, especially when they've learned and applied those facts through genuine problem-solving in which they were able to make choices about strategy based on numbers and problems. Therefore, *Building Fact Fluency* includes both contextualized problems and practice opportunities, (3-Act Math Tasks, Anchor Problems, Contextualized Practice Problems, and Image Talks), and decontextualized problem and practice opportunities (Tool Talks, Number Talks, and Games).

Texas Essential Knowledge and Skills Alignment

– KINDERGARTEN –

"The primary focal areas in Kindergarten are understanding counting and cardinality, understanding addition as joining and subtraction as separating, and comparing objects by measurable attributes.

(A) Students develop number and operations through several fundamental concepts. Students know number names and the counting sequence. Counting and cardinality lay a solid foundation for number. Students apply the principles of counting to make the connection between numbers and quantities.
(B) Students use meanings of numbers to create strategies for solving problems and responding to practical situations involving addition and subtraction.
(C) Students identify characteristics of objects that can be measured and directly compare objects according to these measurable attributes."

The *Building Fact Fluency* toolkit offers abundant resources aligned to two of the three focal areas for Kindergarten: (A) counting and cardinality and (B) addition and subtraction.

Number and operations. The student applies mathematical process standards to understand how to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system. The student is expected to:	
(A) count forward and backward to at least 20 with and without objects	All 94 Image Talks and Tool Talks involve up to 20 objects to count. Number Talks build off the more concrete counting experiences in Image and Tool Talks, encouraging connections between symbolic representations and quantities.
(B) read, write, and represent whole numbers from 0 to at least 20 with and without objects or pictures	The 94 Image and Tool Talks invite students to work with up to 20 objects in a picture, and teachers model representing those objects. In the 31 Number Talks, students work without objects and pictures. In all the problem-based lessons (Anchor Problems, Contextualized Practice Problems, 3-Act Tasks), students read, write, and represent 0–20 and beyond objects as they solve contextualized problems. Games offer opportunities for students to practice reading numbers and connecting representations to numbers (e.g., recognize that 8 dots in a ten frame corresponds to the number 8).
(C) count a set of objects up to at least 20 and demonstrate that the last number said tells the number of objects in the set regardless of their arrangement or order	The 94 Image and Tool Talks each offer students opportunities to count up to 20 objects in different configurations.

(D) recognize instantly the quantity of a small group of objects in organized and random arrangements	Every Image and Tool Talk has 1–20 objects to count in different configurations, including lines (e.g. Marbles, Markers, Bears, Rekenreks, Blocks, Linking Cubes, Lemonade), arrays (e.g. Buttons, Toy Cars, Shells, Apples, Coins, Eggs, Ten Frames), circles (e.g., Pizza, Beads on a ring, Peppers), organized configurations (e.g., Dice and Dominoes) and random arrangements (e.g., Crayons, Tennis Balls, Pattern Block Flowers). These varied groups within 20 encourage subitizing.
(F) generate a number that is one more than or one less than another number up to at least 20	The Plus and Minus 0, 1, & 2 contexts (Shells, Bears, and Blocks) provide several weeks of practice with this standard in all routines, including the Making More and Less game. The Contextualized Practice Problems in these Lesson Strings invite students to extend their understanding beyond 20.
(G) compare sets of objects up to at least 20 in each set using comparative language	Every Image and Tool Talk involves a series of images where students consider what has changed between images. Are there now more or less? How many more/less?
(I) compose and decompose numbers up to 10 with objects and pictures	Almost every task in *Building Fact Fluency* aligns to this standard, particularly those in the Combinations for Ten (Buttons, Markers, Peppers contexts) and Pretend-10/Make-10 (Markers, Pizza, Tennis Balls contexts) strategies. Students are constantly taking numbers apart and putting them back together.
Number and operations. The student applies mathematical process standards to develop an understanding of addition and subtraction situations in order to solve problems. The student is expected to:	
(A) model the action of joining to represent addition and the action of separating to represent subtraction	Students discuss the actions of addition and subtraction in every Image, Tool, and Number Talk, with teachers recording a variety of representations. The hundreds of Anchor Problems, Contextualized Practice Problems, and 3-Act Tasks provide contextualized problem-solving opportunities for students to model the actions in addition and subtraction.
(B) solve word problems using objects and drawings to find sums up to 10 and differences within 10	The Contextualized Practice Problems and Anchor Problems provide ample practice with word problems of every problem type with unknowns in all positions. Students are encouraged to use tools, actions, and representations. Numbers are offered within 5, 10, 20, and multidigit, so students can solve problems of appropriate challenge.

(C) explain the strategies used to solve problems involving adding and subtracting within 10 using spoken words, concrete and pictorial models, and number sentences	Every Problem-Based Lesson (Contextualized Practice Problems, Anchor Problems, and 3-Act Math Tasks) offer opportunities for strategy shares and discussion around student work. In addition, the Image, Tool, and Number Talks give teachers opportunities to model these representations as students share their strategies. The formative assessment strategies recommended in *Building Fact Fluency*—conferences, observations, journaling, and reflection—support teachers listening to these explanations regularly.
Number and operations. The student applies mathematical process standards to identify coins in order to recognize the need for monetary transactions.	
The student is expected to identify U.S. coins by name, including pennies, nickels, dimes, and quarters.	The Coins Lesson String involves pictures of dimes and pennies, along with story problems using these coins.

Texas Essential Knowledge and Skills Alignment

– 1ST GRADE –

"The primary focal areas in Grade 1 are understanding and applying place value, solving problems involving addition and subtraction, and composing and decomposing two-dimensional shapes and three-dimensional solids.

(A) Students use relationships within the numeration system to understand the sequential order of the counting numbers and their relative magnitude.
(B) Students extend their use of addition and subtraction beyond the actions of joining and separating to include comparing and combining. Students use properties of operations and the relationship between addition and subtraction to solve problems. By comparing a variety of solution strategies, students use efficient, accurate, and generalizable methods to perform operations.
(C) Students use basic shapes and spatial reasoning to model objects in their environment and construct more complex shapes. Students are able to identify, name, and describe basic two-dimensional shapes and three-dimensional solids."

The *Building Fact Fluency* toolkit offers abundant resources aligned to two of the three focal areas for first grade: (A) counting, numeration, and magnitude and (B) addition and subtraction.

Number and operations. The student applies mathematical process standards to represent and compare whole numbers, the relative position and magnitude of whole numbers, and relationships within the numeration system related to place value. The student is expected to:	
(A) recognize instantly the quantity of structured arrangements	Every Image and Tool Talk has 1–20 objects to count in different configurations, including lines (e.g. Marbles, Markers, Bears, Rekenreks, Blocks, Linking Cubes, Lemonade), arrays (e.g. Buttons, Toy Cars, Shells, Apples, Coins, Eggs, Ten Frames), circles (e.g., Pizza, Beads on a ring, Peppers), organized configurations (e.g., Dice and Dominoes) and random arrangements (e.g., Crayons, Tennis Balls, Pattern Block Flowers). These varied groups within 20 encourage subitizing.
Number and operations. The student applies mathematical process standards to develop and use strategies for whole number addition and subtraction computations in order to solve problems. The student is expected to:	
(B) use objects and pictorial models to solve word problems involving joining, separating, and comparing sets within 20 and unknowns as any one of the terms in the problem such as 2 + 4 = ☐; 3 + ☐ = 7; and 5 = ☐ - 3	The Contextualized Practice Problems and Anchor Problems provide ample practice with word problems of every problem type, with unknowns in all positions. Numbers are offered within 5, 10, 20, and multidigit so students can solve problems of appropriate challenge. Students represent the operations with objects, drawings, and equations.
(C) compose 10 with two or more addends with and without concrete objects	The Buttons (4), Markers (8), and Peppers (10) Lesson Strings are specifically focused on decomposing and composing ten. Throughout the toolkit, use of ten-frames, linking cubes, Base 10 blocks, and Rekenreks encourage students to explore ten.

(D) apply basic fact strategies to add and subtract within 20, including making 10 and decomposing a number leading to a 10	Every single task in the *Building Fact Fluency* toolkit aligns to this standard, including many weeks devoted to both the Pretend-10/Make-10 and Combinations for 10 strategies.
(E) explain strategies used to solve addition and subtraction problems up to 20 using spoken words, objects, pictorial models, and number sentences	Every single task in the *Building Fact Fluency* toolkit aligns to this standard.
(G) generate and solve problem situations when given a number sentence involving addition or subtraction of numbers within 20	The Number Talks follow the Image and Tool Talks, so students are invited to connect the symbolic problem in the Number Talk back to the contexts discussed earlier in the Lesson String. Or, they might generate new problem contexts.
Number and operations. The student applies mathematical process standards to identify coins, their values, and the relationships among them in order to recognize the need for monetary transactions. The student is expected to:	
(A) identify U.S. coins, including pennies, nickels, dimes, and quarters, by value and describe the relationships among them	The Coins Lesson String involves pictures of dimes and pennies, along with story problems using these coins.
(B) write a number with the cent symbol to describe the value of a coin	The Coins context involves pictures of dimes and pennies, along with story problems using these coins.
Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:	
(D) represent word problems involving addition and subtraction of whole numbers up to 20 using concrete and pictorial models and number sentences	All the problem-based tasks in the toolkit—Anchor Problems, Contextualized Practice Problems, and 3-Act Tasks—give students opportunities to solve word problems of every type using objects, representations, and equations.
(E) understand that the equal sign represents a relationship where expressions on each side of the equal sign represent the same value(s)	The connections from Image to Tool to Number Talk invite much discussion about the symbolic representation of addition and subtraction. In addition, there are many opportunities to compare expressions, leading to equations such as 5 + 3 = 3 + 5 and 7 + 3 = 8 + 2, which invite relational thinking about the equals sign (as opposed to thinking the equals sign means “the answer comes next”).
(F) determine the unknown whole number in an addition or subtraction equation when the unknown may be any one of the three or four terms in the equation	The Contextualized Practice Problems provide ample practice opportunities for every problem type, with unknowns in all positions.
(G) apply properties of operations to add and subtract two or three numbers	All tasks in *Building Fact Fluency* are designed so students will discover, explore, and understand the properties of operations in addition and subtraction.

Texas Essential Knowledge and Skills Alignment

– 2ND GRADE –

"The primary focal areas in Grade 2 are making comparisons within the base-10 place value system, solving problems with addition and subtraction within 1,000, and building foundations for multiplication.

(A) Students develop an understanding of the base-10 place value system and place value concepts. The students' understanding of base-10 place value includes ideas of counting in units and multiples of thousands, hundreds, tens, and ones and a grasp of number relationships, which students demonstrate in a variety of ways.
(B) Students identify situations in which addition and subtraction are useful to solve problems. Students develop a variety of strategies to use efficient, accurate, and generalizable methods to add and subtract multi-digit whole numbers.
(C) Students use the relationship between skip counting and equal groups of objects to represent the addition or subtraction of equivalent sets, which builds a strong foundation for multiplication and division."

The *Building Fact Fluency* toolkit offers abundant resources aligned to one of the three focal areas for second grade: (B) addition and subtraction.

Number and operations. The student applies mathematical process standards to develop and use strategies and methods for whole number computations in order to solve addition and subtraction problems with efficiency and accuracy. The student is expected to:	
(A) recall basic facts to add and subtract within 20 with automaticity	Every single task in *Building Fact Fluency* is designed to build fluency within 20 while building conceptual understanding of addition and subtraction.
(B) solve one-step and multi-step word problems involving addition and subtraction within 1,000 using a variety of strategies based on place value, including algorithms	The multi-digit options of the Contextualized Practice Problems and Anchor Problems offer hundreds of opportunities to practice adding and subtracting within 100 using strategies based on place value, the properties, and relationships between the operations. Teachers could certainly choose bigger numbers to substitute.
Algebraic reasoning. The student applies mathematical process standards to identify and apply number patterns within properties of numbers and operations in order to describe relationships. The student is expected to:	
(C) represent and solve addition and subtraction word problems where unknowns may be any one of the terms in the problem	The *Building Fact Fluency* problem-based lessons—Contextualized Practice Problems, Anchor Problems, and 3-Act Tasks—provide hundreds of opportunities for students to solve addition and subtraction word problems within 100 in all problem types and with unknowns in all positions. The variety of problem types encourages relational thinking. For example, a "join change unknown" problem can be thought of as 4 + ? = 10, or as 10 - 4 = ?, which generates thinking about the relationship between the operations of addition and subtraction. Students' representations of their work are core to these lessons and to formative assessment within the toolkit.

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