

Building Fact Fluency

A TOOLKIT FOR ADDITION & SUBTRACTION

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CORRELATION TO

**Florida's B.E.S.T. Standards –
Mathematical Thinking
and Reasoning Standards**

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Florida’s B.E.S.T. Standards – Mathematical Thinking and Reasoning Standards

<p>MA.K12.MTR.1.1: Actively participate in effortful learning both individually and collectively.</p>	<p>Individual and collaborative problem-solving is embedded throughout <i>Building Fact Fluency</i>, especially in these routines:</p> <ul style="list-style-type: none"> · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Journaling and Reflection <p>In addition, the structure of <i>Building Fact Fluency</i> provides frequent and regular opportunities for genuine problem-solving with collaboration and feedback, building students’ problem-solving skills and stamina.</p>
<p>MA.K12.MTR.2.1: Demonstrate understanding by representing in multiple ways.</p>	<p>Student representations are embedded throughout <i>Building Fact Fluency</i>, especially in problem-based lesson discussions and the following routines:</p> <ul style="list-style-type: none"> · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Reflection and Journaling <p>Teachers model a variety of representations in these warm-up routines:</p> <ul style="list-style-type: none"> · Image Talks · Tool Talks · Number Talks <p>The Lesson Strings support connections across representations and contexts, e.g., from everyday objects to mathematical tools to numeric representation.</p>
<p>MA.K12.MTR.3.1: Complete tasks with mathematical fluency.</p>	<p>The <i>Building Fact Fluency</i> toolkit is designed so students develop powerful single-digit and operational fluency alongside and intertwined with conceptual understanding. The goal is that students will ultimately both know and be able to use their facts as they solve problems with flexibility, accuracy, and efficiency. Every task is engineered toward this goal.</p>
<p>MA.K12.MTR.4.1 Engage in discussions that reflect on the mathematical thinking of self and others.</p>	<p>Mathematical communication, analysis, justifications, and argumentation are embedded throughout <i>Building Fact Fluency</i>, especially in problem-based lesson discussions, student representations, and the following discussion-based components:</p> <ul style="list-style-type: none"> · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Games · Interviews, conferences, journaling, and reflection

<p>MA.K12.MTR.5.1 Use patterns and structure to help understand and connect mathematical concepts.</p>	<p><i>Building Fact Fluency</i> is designed so students look for structure and discover the properties of the operations to learn their number combinations, rather than memorize facts in isolation. Therefore, there are many opportunities to analyze relationships, seek patterns, look for structure, connect ideas, and make generalizations across representations, contexts, problems, and strategies throughout the toolkit, especially in these routines:</p> <ul style="list-style-type: none"> · Image Talks · Tool Talks · Number Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks · Games <p>The problem-based lessons (Anchor Problems, Contextualized Practice Problems, and 3-Act Math Tasks) give students opportunities to solve larger and more complex problems.</p>
<p>MA.K12.MTR.6.1 Assess the reasonableness of solutions.</p>	<p>The <i>Building Fact Fluency</i> toolkit is designed to build number sense and mathematical sensemaking. All tasks give students opportunities to develop an intuitive sense of the magnitude of numbers and how the operations work. The problem-based lessons (Anchor Problems, Contextualized Practice Problems, and 3-Act Math Tasks) feature estimation and reasonableness explicitly. Students explain their solution methods across all components of the Lesson Strings.</p>
<p>MA.K12.MTR.7.1 Apply mathematics to real-world contexts.</p>	<p>The <i>Building Fact Fluency</i> toolkit is based on everyday contexts, e.g., crayons, shells, marbles, buttons, markers, bears, toy cars, lemonade, peppers, pizza, apples, blocks, coins, eggs, chopsticks, and tennis balls. Students solve contextualized problems in these routines in every Lesson String:</p> <ul style="list-style-type: none"> · Image Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks <p>All the problem-based lessons (especially the 3-Act Math Tasks) give students opportunities to model with mathematics.</p>

Fluency with Arithmetic Operations and Automaticity with Basic Arithmetic Facts

Throughout [the B.E.S.T. standards], benchmark expectations regarding arithmetic operations within the Number Sense and Operations (NSO) strand have been developed with a hierarchy in mind consisting of three stages: exploration, procedural reliability and procedural fluency. Students will first explore arithmetic operations with no fluency expectations, then will be able to show procedural reliability and finally they will carry out these operations with procedural fluency. Interwoven into this hierarchy is the development of direct recall of basic arithmetic facts. Basic arithmetic facts are first derived, then utilized while becoming procedurally reliable or fluent and finally recalled with automaticity.

- **Stage 1: Exploration.** The expectation is to develop understanding through the use of manipulatives, visual models, discussions, estimation and drawings.
- **Stage 2: Procedural reliability.** The expectation is to utilize skills from the exploration stage to develop an accurate, reliable method that aligns with the student's understanding and learning style. Students may need the teacher's help to choose a method, and they will learn how to use a method without help.
- **Stage 3: Procedural fluency.** The expectation is to utilize skills from the procedural reliability stage to become fluent with an efficient and accurate procedure, including a standard algorithm
- **Embedded within Stage 1 - Stage 3: Automaticity.** The expectation is to directly recall basic arithmetic facts from memory. Automaticity is the ability to act according to an automatic response which is easily retrieved from long-term memory. It usually results from repetition and practice.

The *Building Fact Fluency* toolkit encourages fact fluency development through the stages referred to in the B.E.S.T. fluency standards. While students' progression through these phases is not linear and varies according to the problem, context, numbers, etc., students generally move from direct modeling through counting strategies toward deriving strategies. By using what they know to figure out things they don't know, students uncover the properties of the operations, develop algorithms and strategies, and begin to "just know" some facts. Over time, students with sufficient opportunities to practice can retrieve most facts from memory in the midst of problem solving. Therefore, this toolkit is designed to give students ample experience exploring and then deriving addition and subtraction number combinations through a variety of representations, contexts, and problem types. Fluency is an outcome of this problem solving and purposeful practice.

From the Facilitator's Guide:

"The most important thing to keep in mind when thinking about fluency work is that the turtle usually wins the race. Remember that our historic rush to memorize facts ignores how students really learn their number combinations with deep understanding. We *cannot* just jump from counting to memorization...Instead, we must spend plenty of time lingering in modeling, counting, and deriving, where students are developing number sense, thinking about relationships, and uncovering the properties of the operations. Eventually, all that rich work pays off, when we see confident students who have fluency with their number combinations and a rock-solid foundation for number, operations, and algebraic thinking."

Required Procedural Reliability, Procedural Fluency and Basic Fact Automaticity for the Operations: Addition and Subtraction

<p>Kindergarten</p>	<p>Procedural Reliability: Two one-digit whole numbers with sums from 0 to 10 and related subtraction facts</p>	<p><i>Building Fact Fluency</i> provides differentiated opportunities for students to work with their single digit facts within 5, within 10, within 20, and with multi-digit numbers. The Sums within 5; Plus and Minus 0, 1, 2; Doubles; and Combinations for 10 strategies are particularly well suited for the B.E.S.T. kindergarten fluency expectations, and the remaining strategies (10 and Some More, Near Doubles, and Pretend-10/Make-10) offer opportunities for exploration of strategies to come.</p>
<p>First grade</p>	<p>Recall: Two whole numbers with sums from 0 to 10 and related subtraction facts</p> <p>Procedural Reliability: Two whole numbers with sums from 0 to 20 and related subtraction facts</p>	<p><i>Building Fact Fluency</i> provides differentiated opportunities for students to work with their single digit facts within 5, within 10, within 20, and with multidigit numbers. Therefore, first graders will have plenty of opportunities to develop fluency with their addition and subtraction facts within 10 and continue developing and extending strategies for their facts within 20, while they explore strategies for multi-digit problems.</p>
<p>Second grade</p>	<p>Recall: Two whole numbers with sums from 0 to 20 and related subtraction facts</p> <p>Procedural Reliability: Two whole numbers with sums up to 100 and subtract a whole number from a whole number, each no larger than 100</p>	<p><i>Building Fact Fluency</i> provides differentiated opportunities for students to work with their single digit facts within 5, within 10, within 20, and with multidigit numbers. The toolkit is designed such that, by the end of second grade, most students will know their facts within 20 if they've had sufficient opportunities to practice. In addition, the multi-digit options in the Contextualized Practice Problems encourage extension of the understanding developed with single-digit facts into multi-digit numbers within 100.</p>

Florida’s B.E.S.T. Standards – Mathematical Thinking and Reasoning Standards – KINDERGARTEN –

“In Kindergarten, instructional time will emphasize three areas:

- (1) developing an understanding of counting to represent the total number of objects in a set and to order the objects within a set;
- (2) developing an understanding of addition and subtraction and the relationship of these operations to counting and
- (3) measuring, comparing and categorizing objects according to various attributes, including their two- and three-dimensional shapes.”

The *Building Fact Fluency* toolkit offers abundant resources aligned to two of the three focal areas for Kindergarten:

- (1) counting and (2) addition and subtraction.

Number Sense and Operations	
MA.K.NSO.1 Develop an understanding for counting using objects in a set.	
MA.K.NSO.1.1 Given a group of up to 20 objects, count the number of objects in that group and represent the number of objects with a written numeral. State the number of objects in a rearrangement of that group without recounting.	All 94 Image Talks and Tool Talks involve series of images with up to 20 objects in different configurations to count and mentally rearrange. Number Talks build off the more concrete counting experiences in Image and Tool Talks, encouraging connections between symbolic representations and quantities.
MA.K.NSO.1.2 Given a number from 0 to 20, count out that many objects.	In all the problem-based lessons (Anchor Problems, Contextualized Practice Problems, 3-Act Tasks), kindergarten students will model problems with objects or represent them in writing as they solve contextualized problems. Therefore, there are many opportunities for students to count objects within the context of problem solving.
MA.K.NSO.1.4 Compare the number of objects from 0 to 20 in two groups using the terms less than, equal to or greater than.	Every Image and Tool Talk involves a series of 3-6 images where students consider what has changed between images. Are there now more or less? How many more/less?
MA.K.NSO.2 Recite number names sequentially within 100 and develop an understanding for place value.	
MA.K.NSO.2.1 Recite the number names to 100 by ones and by tens. Starting at a given number, count forward within 100 and backward within 20.	Students have opportunities to develop understanding of place value and numbers within 100 in many tasks in the <i>Building Fact Fluency</i> toolkit, especially in the Contextualized Practice Problems, which provide number choices within 5, within 10, within 20, and with multi-digit numbers to 100. Recitation is not an explicit component of the kit, but students will make connections between counting and addition and subtraction and explore place value by composing and decomposing tens within meaningful contexts.
MA.K.NSO.2.2 Represent whole numbers from 10 to 20, using a unit of ten and a group of ones, with objects, drawings and expressions or equations.	The Markers (5), Pizza (11), and Coins (15) Lesson Strings explicitly focus on the “Ten and Some More” numbers, including “The Teen Game.” The Games and Tool Talks in the Ten and Some More Lesson Strings include multiple representations and mathematical tools, such as ten-frames and loose ones, double-ten frames, Base-10 blocks, linking cubes and Rekenreks.

MA.K.NSO.3 Develop an understanding of addition and subtraction operations with one-digit whole numbers.	
MA.K.NSO.3.1 Explore addition of two whole numbers from 0 to 10, and related subtraction facts.	Every single task in <i>Building Fact Fluency</i> aligns to this standard.
MA.K.NSO.3.2 Add two one-digit whole numbers with sums from 0 to 10 and subtract using related facts with procedural reliability.	Every single task in <i>Building Fact Fluency</i> aligns to this standard.
Algebraic Reasoning	
MA.K.AR.1 Represent and solve addition problems with sums between 0 and 10 and subtraction problems using related facts.	
MA.K.AR.1.1 For any number from 1 to 9, find the number that makes 10 when added to the given number.	The Buttons (4), Markers (8), and Peppers (10) Lesson Strings are explicitly focused on the “Combinations for Ten” strategy, with plenty of embedded practice in all routines. The two “Partners for Ten” games specifically target this standard.
MA.K.AR.1.2 Given a number from 0 to 10, find the different ways it can be represented as the sum of two numbers.	Both the Sums within 5 [Crayons (1), Marbles (3), and Toy Cars (7)] and Combinations for 10 [Buttons (4), Markers (8), and Peppers (10)] Lesson Strings emphasize decomposing and composing numbers within 10. For example, the Peppers Image Talk shows 10 peppers on two plates: 6 red and 4 yellow, then 4 red and 6 yellow, 7 red and 3 yellow, 3 red and 7 yellow, and so on. Teachers record representations and equations. In addition, the Contextualized Practice Problems include “both addends unknown” problem types in which students will find all the combinations to make a particular number.
MA.K.AR.1.3 Solve addition and subtraction real-world problems using objects, drawings or equations to represent the problem.	The <i>Building Fact Fluency</i> toolkit is based on everyday contexts, e.g., crayons, shells, marbles, buttons, markers, bears, toy cars, lemonade, peppers, pizza, apples, blocks, coins, eggs, chopsticks, and tennis balls. Students solve contextualized problems in these routines: <ul style="list-style-type: none"> · Image Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks The problems are organized by problem type, so teachers can easily select problems aligned with state expectations for Kindergarten in the “Situations Involving Operations with Numbers” document.
MA.K.AR.2 Develop an understanding of the equal sign.	
MA.K.AR.2.1 Explain why addition or subtraction equations are true using objects or drawings.	The connections from Image to Tool to Number Talk invite much discussion about the symbolic representation of addition and subtraction. In addition, there are many opportunities to compare expressions, leading to equations such as $5 + 3 = 3 + 5$ and $7 + 3 = 8 + 2$, which invite relational thinking about the equals sign (as opposed to thinking the equals sign means “the answer comes next”). Students are expected to justify their solutions with objects and drawings across the <i>Building Fact Fluency</i> routines.

Florida’s B.E.S.T. Standards – Mathematical Thinking and Reasoning Standards – 1ST GRADE –

“In grade 1, instructional time will emphasize four areas:

- (1) understanding the place value of tens and ones within two-digit whole numbers;
- (2) extending understanding of addition and subtraction and the relationship between them;
- (3) developing an understanding of measurement of physical objects, money and time and;
- (4) categorizing, composing and decomposing geometric figures.”

The *Building Fact Fluency* toolkit offers abundant resources aligned to two of the four focal areas for first grade:

- (1) place value of tens and ones and (2) addition and subtraction.

Number Sense and Operations	
MA.1.NSO.2 Develop an understanding of addition and subtraction operations with one- and two-digit numbers.	
MA.1.NSO.2.1 Recall addition facts with sums to 10 and related subtraction facts with automaticity.	The <i>Building Fact Fluency</i> toolkit was written to support development of fact fluency alongside and intertwined with conceptual understanding of addition and subtraction. Every task aligns to this standard, and there are ample opportunities for first graders to build fluency within 10.
MA.1.NSO.2.2 Add two whole numbers with sums from 0 to 20, and subtract using related facts with procedural reliability.	The <i>Building Fact Fluency</i> toolkit was written to support development of fact fluency alongside and intertwined with conceptual understanding of addition and subtraction. Every task aligns to this standard, and there are plenty of opportunities for first graders to use a variety of strategies to solve problems within 20 as they become increasingly fluent.
MA.1.NSO.2.3 Identify the number that is one more, one less, ten more and ten less than a given two-digit number.	The Plus and Minus 0, 1, & 2 Lesson Strings [Shells (2), Bears (6), and Blocks (13)] provide several weeks of practicing one more and less in all routines, including the Making More and Less game. The Contextualized Practice Problems in these Lesson Strings invite students to extend their understanding beyond 20, and discussion among students solving a range of number choices, (e.g. $4 + 6$, $14 + 6$, $24 + 6$) would yield discovery of patterns around ten more and ten less.
MA.1.NSO.2.4 Explore the addition of a two-digit number and a one-digit number with sums to 100.	The Contextualized Practice Problems and Anchor Problems provide ample practice with addition and subtraction word problems of every problem type with unknowns in all positions. Students are encouraged to use tools, objects, and representations. Numbers are offered within 5, 10, 20, and multidigit within 100, so students can solve problems of appropriate challenge. The problems are organized by problem type, so teachers can easily select problems aligned with state expectations for 1st Grade in the “Situations Involving Operations with Numbers” document.
MA.1.NSO.2.5 Explore subtraction of a one-digit number from a two-digit number.	The Contextualized Practice Problems and Anchor Problems provide ample practice with addition and subtraction word problems of every problem type with unknowns in all positions. Students are encouraged to use tools, objects, and representations. Numbers are offered within 5, 10, 20, and multidigit, within 100, so students can solve problems of appropriate challenge. The problems are organized by problem type, so teachers can easily select problems aligned with state expectations for 1st Grade in the “Situations Involving Operations with Numbers” document.

Algebraic Reasoning**MA.1.AR.1** Solve addition problems with sums between 0 and 20 and subtraction problems using related facts.

MA.1.AR.1.1 Apply properties of addition to find a sum of three or more whole numbers.

The problems in *Building Fact Fluency* tend to have two addends, but students' strategies or the way the objects are organized in the tasks often invite students to decompose those numbers such that they work with multiple addends, especially in the Near Doubles or Pretend-10/Make-10 strategies (e.g., $8 + 7$ can be thought of as $7 + 7 + 1$ or $8 + 2 + 5$).

MA.1.AR.1.2 Solve addition and subtraction real-world problems using objects, drawings or equations to represent the problem.

The *Building Fact Fluency* toolkit is based on everyday contexts, e.g., crayons, shells, marbles, buttons, markers, bears, toy cars, lemonade, peppers, pizza, apples, blocks, coins, eggs, chopsticks, and tennis balls. Students solve contextualized problems in these routines in every Lesson String:

- Image Talks
- Contextualized Practice Problems
- Anchor Problems
- 3-Act Math Tasks

MA.1.AR.2 Develop an understanding of the relationship between addition and subtraction.

MA.1.AR.2.1 Restate a subtraction problem as a missing addend problem using the relationship between addition and subtraction.

The Contextualized Practice Problems and Anchor Problems provide ample practice with word problems of every problem type, so students are joining, separating, comparing, and analyzing parts and wholes. The variety of problem types encourages relational thinking. For example, a "join change unknown" problem can be thought of as $4 + ? = 10$, or as $10 - 4 = ?$. The problems are organized by problem type, so teachers can easily select problems aligned with state expectations for 1st Grade in the "Situations Involving Operations with Numbers" document.

MA.1.AR.2.2 Determine and explain if equations involving addition or subtraction are true or false.

The connections from Image to Tool to Number Talk invite much discussion about the symbolic representation of addition and subtraction. In addition, there are many opportunities to compare expressions, leading to equations such as $5 + 3 = 3 + 5$ and $7 + 3 = 8 + 2$, which invite relational thinking about the equals sign (as opposed to thinking the equals sign means "the answer comes next").

MA.1.AR.2.3 Determine the unknown whole number in an addition or subtraction equation, relating three whole numbers, with the unknown in any position.

The *Building Fact Fluency* problem-based lessons--Contextualized Practice Problems, Anchor Problems, and 3-Act Tasks--provide hundreds of opportunities for students to solve addition and subtraction word problems within 100 in all problem types and with unknowns in all positions. The problems are organized by problem type, so teachers can easily select problems aligned with state expectations for 1st Grade in the "Situations Involving Operations with Numbers" document.

Measurement**MA.1.M.2** Tell time and identify the value of coins and combinations of coins and dollar bills.

MA.1.M.1.1 Identify pennies, nickels, dimes and quarters, and express their values using the ¢ symbol. State how many of each coin equal a dollar.

The Coins Lesson String involves pictures of dimes and pennies, along with story problems using these coins and written representations of the ¢ symbol.

Florida’s B.E.S.T. Standards – Mathematical Thinking and Reasoning Standards – 2ND GRADE –

Florida B.E.S.T - 2ND GRADE

“In grade 2, instructional time will emphasize four areas:

- (1) extending understanding of place value in three-digit numbers;
- (2) building fluency and algebraic reasoning with addition and subtraction;
- (3) extending understanding of measurement of objects, time and the perimeter of geometric figures and
- (4) developing spatial reasoning with number representations and two-dimensional figures.”

The *Building Fact Fluency* toolkit offers abundant resources aligned to one of the four focal areas for second grade:

- (1) addition and subtraction.

Number sense and operations	
Add and subtract two and three-digit whole numbers.	
MA.2.NSO.2.1 Recall addition facts with sums to 20 and related subtraction facts with automaticity.	Every single task in <i>Building Fact Fluency</i> is designed to build fluency within 20 while building conceptual understanding of addition and subtraction.
MA.2.NSO.2.3 Add two whole numbers with sums up to 100 with procedural reliability. Subtract a whole number from a whole number, each no larger than 100, with procedural reliability.	The multi-digit options of the Contextualized Practice Problems and Anchor Problems offer hundreds of opportunities to practice adding and subtracting within 100 using strategies based on place value, the properties, and relationships between the operations.
Algebraic reasoning	
MA.2.AR.1 Solve addition problems with sums between 0 and 100 and related subtraction problems.	
MA.2.AR.1.1 Solve one- and two-step addition and subtraction real-world problems.	<p>The <i>Building Fact Fluency</i> toolkit is based on everyday contexts, e.g., crayons, shells, marbles, buttons, markers, bears, toy cars, lemonade, peppers, pizza, apples, blocks, coins, eggs, chopsticks, and tennis balls. Students solve contextualized problems in these routines in every Lesson String:</p> <ul style="list-style-type: none"> · Image Talks · Contextualized Practice Problems · Anchor Problems · 3-Act Math Tasks <p>These problems provide hundreds of opportunities for students to solve addition and subtraction word problems within 100 in all problem types and with unknowns in all positions. The problems are organized by problem type, so teachers can easily select problems aligned with state expectations for 2nd Grade in the “Situations Involving Operations with Numbers” document.</p>

MA.2.AR.2 Demonstrate an understanding of equality and addition and subtraction.	
MA.2.AR.2.1 Determine and explain whether equations involving addition and subtraction are true or false.	The connections from Image to Tool to Number Talk invite much discussion about the symbolic representation of addition and subtraction. In addition, there are many opportunities to compare expressions, leading to equations such as $5 + 3 = 3 + 5$ and $7 + 3 = 8 + 2$, which invite relational thinking about the equals sign (as opposed to thinking the equals sign means “the answer comes next”). Students are expected to justify their solutions
MA.2.AR.2.2 Determine the unknown whole number in an addition or subtraction equation, relating three or four whole numbers, with the unknown in any position.	The <i>Building Fact Fluency</i> problem-based lessons--Contextualized Practice Problems, Anchor Problems, and 3-Act Tasks--provide hundreds of opportunities for students to solve addition and subtraction word problems within 100 in all problem types and with unknowns in all positions. The problems are organized by problem type, so teachers can easily select problems aligned with state expectations for 2nd Grade in the “Situations Involving Operations with Numbers” document.
Measurement	
MA.2.M.2 Tell time and solve problems using money.	
MA.2.M.2.2 Solve one- and two-step addition and subtraction real-world problems involving either dollar bills within \$100 or coins within 100¢ using \$ and ¢ symbols appropriately.	The Coins Lesson String involves pictures of dimes and pennies, along with story problems using these coins and written representations of the ¢ symbol.



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