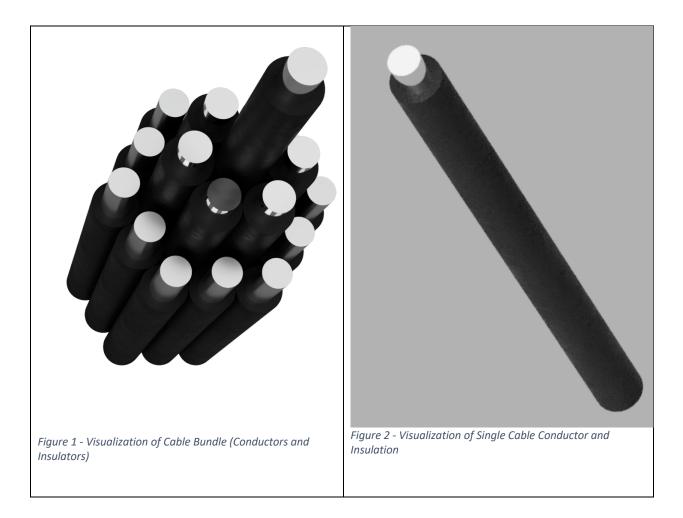
POWER LOSS COMPARISON FOR LARGE DIAMETER (~1") DC CABLES SINGLE CABLE VS. BUNDLE



The problem presented is to determine whether a bundle of cables experiences more energy loss due to Joule heating vs. individual/isolated cables, and quantify that energy loss to determine the viability of improved efficiency through isolation and separation of cables in a bundle.

Modeling the Thermal Behavior

The approach taken to perform the analysis is based in the Steady State solution, meaning rates of energy transfer have achieved their steady behavior.

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\binom{Rate \ of \ Heat}{Into \ Object} = \binom{Rate \ of \ Heat}{Out \ of \ Object}
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This is referred to as Fourier's law of heat conduction, and is given by:

$$\dot{Q} = -kA \frac{dT}{dr}$$
 for a cylinder

Which may be integrated:

$$Q = \int_0^{\Delta t} Q^* dt$$

The result may be simplified by using the 'resistor model' approach, namely the energy loss terms may be written as resistance elements which are summed to form a total resistance value.

$$Q^* = \frac{(T_1 - T_\infty)}{R_{total}}$$

The resistive losses are effectively the energy lost in convection (heat transfer to the environment/atmosphere)

$$R_{convect} = \frac{1}{h \cdot A}$$

And the energy transfered to the insulation

$$R_1 = R_{insulation} = \frac{\ln \left(r_2 / r_1 \right)}{2\pi k_1 L}$$

Which may be combined to provide the total resistance.

$$R_{total} = \sum R = R_{convect} + R_{insulation} = \frac{1}{h \cdot A} + \frac{\ln (r_2/r_1)}{2\pi k_1 L}$$

As an example, the temperature increase in the conductor of a specific cable has been calculated. Using dimensional specifications for a WTEC A0600BQBBX3 cable:

$$d_{overall} = 1083 mils$$

 $x_{insulation} = 135 mils$

Together with common values for the conductor (Aluminum):

$$\begin{split} \rho &= 2.82 \times 10^{-8} \Omega m \ (resistivity) \\ \partial &= 2.7 \ {gr} / {cm^3} \ (density) \\ k &= 2.37 \ {W} / {mK} \ (thermal \ conductivity) \end{split}$$

and some other values:

$$\begin{split} h &= 10 \frac{W}{m^2 K} \; (convection \; coef \; of \; large \; cylinder \; in \; air) \\ T_0 &= \; 30^o C \; (ambient \; temperature) \\ I &= \; 400A \; (current \; in \; a \; single \; conductor) \end{split}$$

The key calculations involved are (a) the outer surface area of the cable and (b) the ratio of the insulation radius to the conductor radius. For our example, we find that

$$R_{convect} = \frac{1}{hA} = \frac{1}{h(2\pi r_{insulation}L)} = 0.779 \, K/W$$

And that

$$R_{insulation} = \frac{\ln (r_2/r_1)}{2\pi kL} = 0.236 \text{K/W}$$

Given that the power input is that due to Joule heating, we find that

$$\dot{Q} = I^2 R$$

where R is the conduction resistance of the material used (in this case Aluminum).

$$R = \frac{\rho}{LA}$$

In this case, A is the cross-sectional area of the conductor, L is the length of the conductor, and ρ is the resistivity of the conducting material. As is expected, the electrical resistance to current flow depends on the volume of material through which the current is passed (consistent with the fact that larger wires can carry more current).

Sticking with our example, we find that $\dot{Q} = 7.13$ W, and we can solve for the unknown temperature term in the following equation:

$$Q^* = \frac{(T_1 - T_{\infty})}{R_{total}} = 7.13W = \frac{(T_1 - T_{\infty})}{(.779K/W + .236K/W)} \text{ and } T_{\infty} = T_{ambient} = 30C$$

This leads to a steady state solution for T_1 of 37.19°C

Given that we can now calculate the temperature of the conductor, we can now compare temperatures for different cable configurations (dimensions and currents). Given the total resistance (R_{total}), we can also compare how cable configurations compare in terms of their respective power loss. The approach used is to look at the temperature dependence of a resistor to calculate an updated resistance value for a given configuration.

$$R = \frac{\rho L}{A} [1 + \alpha (T - T_0)]$$

where α is the conductor's temperature dependence and T_0 is the temperature for that reference value. Then we can re-apply

$$\dot{Q} = I^2 R$$

to find the Joule heating energy lost. We can calculate this value for varous cable configurations and compare them to the base configuration for a single cable. It is important to compare apples with apples, so when we perform the calculation for a 10-conductor 'fat cable', we must compare the result to 10 individual/separated cables that would be performing the same task.

The key arguments required to make the Fourier steady state approach work are (a) that we assume steady state, and (b) we assume symmetry so that the transfer of energy is always radial. To achieve a 'symmetric model' of the bundle, we approximate the bundle of discrete cables (conductor plus insulation) numerically as a single conductor with insulation surrounding it with equivalent cross-sectional areas of both the conductor and the insulation.

Modeling the Bundle vs. a Single Cable

The next step in comparing conductor temperatures between individual cables and bundled cables is to apply the model to a bundle. The primary challenge with the bundle is to find a way to take the radially asymetric distribution of conductor and insulator and make it symetric. Our choice in doing this is to simply focus on a single cable (in this case the center one in Figure 3) and better define what its world looks like. Effectively, the center conductor sees all the insulation from all the other conductors surrounding it. Figure 5 illustrates the 'effective insulation thickness' from a four cable bundle where we make the cross-sectional area of the insulation layer for the center conductor grow to have the same cross-sectional area of the N conductors in the bundle (in this case, four times the insulation).

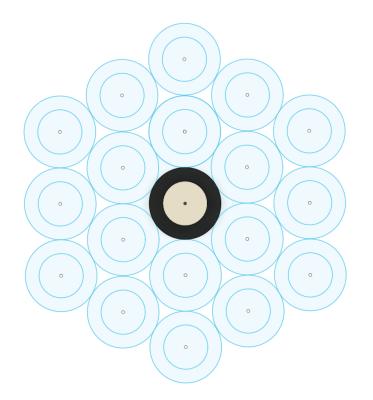
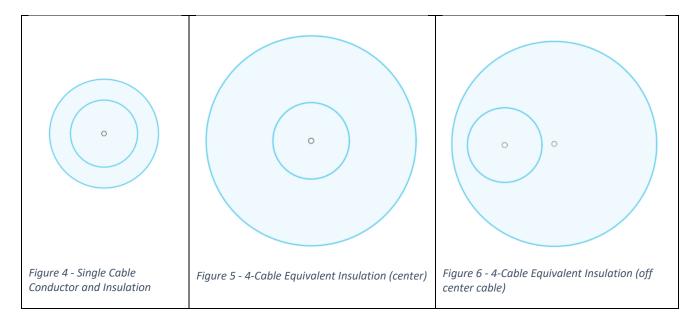


Figure 3 - Center cable in a bundle



We can argue that a cable on the edge of the bundle will effectively behave the same as the center cable since the asymetry leads to more insulation on one side vs. the other, with the net effect being an 'average' thickness of insulation that is the same as that for the center cable.

The next step in the approximation is to consolidate the N conductors each pushing equal amounts of current to one another. Each of these can be viewed as a source for Joule heating, and each of the cables in the bundle will be transfering energy back and forth to one another until that energy gets balanced by the energy transfered to the insulation along with the energy convected away. From Figure 5 and Figure 6, the equivalence is clear since the off-center conductor will see more insulation in one direction than the other, but on average the amount of insulation is the same for both center and off-center.

From this interpretation, we take the next step to combine the N discrete cables into a single monolithic cable that has the same conductor cross section as the sum of the N cables, surrounded by the net cross-sectional area of the insulation. Numerically, this 'fat cable' also carries N times the current of that carried by an individual cable.

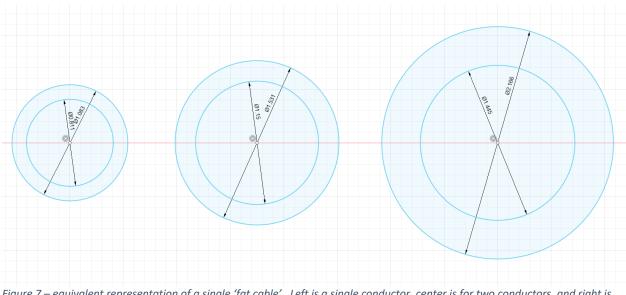


Figure 7 – equivalent representation of a single 'fat cable'. Left is a single conductor, center is for two conductors, and right is for four conductors. The conductive area for four conductors is four times that of a single cable, and the current carried by the 'fat cable' is four times that of the single cable.

Thermal Results

For our example case of the WTEC A0600BQBBX3 cable, we can now calculate the conductor temperature as well as the insulation (surface) temperature.

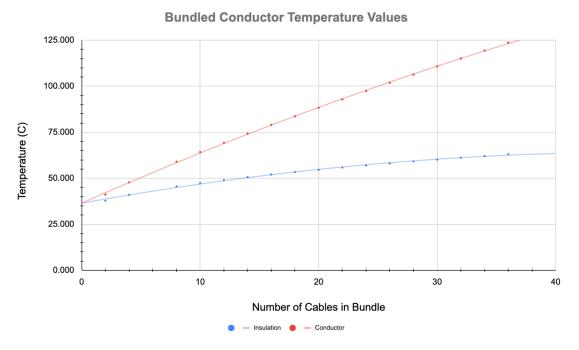


Figure 8 – Conductor temperature (red) and insulation surface temperature (blue) for the 'fat cable' approximation using increasing bundle sizes. Calculation is performed for a 1m cable length.

If we perform the described analysis for relative power loss due to increased heat (vs. single discrete cables), we can find the relative energy loss for bundles of cables vs. assemblies of discrete cables.

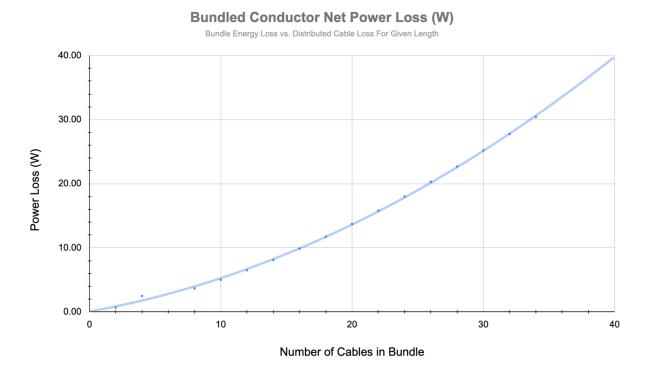


Figure 9 – Bundled power loss. The power loss is calculated in comparison to that of a single/discrete cable. For a 10-cable bundle, the power lost to Joule heating is compared to the power lost by 10 individual cables. Calculations are for a 1m cable length.

It turns out that the above chart is specific to our example cable (WTEC A0600BQBBX3), and is calculated for a 1m cable length. Thus, the values in the figure can be used as loss-per-meter since a 100m long bundle will have 100 times the loss.

Watts and Kilowatt Hours

For most of us, energy use and energy costs are based on the Kilowatt-hour (kWh). Energy companies charge for electricity usage by the kWh, not by the Watt. To convert our results to something monetary, we need to understand how energy usage.

For example, if the electrical company charges \$0.11/kWh, then a 100W lightbulb that is on for 8 hours is consuming 800Wh or 0.8kWh. Powering this bulb for three hours will cost 0.8kWh*\$0.11/kWh or \$0.09.

From Figure 9 we can see that a 30-conductor cable loses approximately 25W more per meter than the same 30 conductors configured in a manner where they don't transfer heat to one another. That means that for a 100m configuration, that bundle will lose 250W more. Over the course of 12 hours, that translates to 3000Wh or 3kWh. Over the course of a year, that becomes 1095kWh.