Waring School Summer Math Packet

for students entering Precalculus

Hello folks,

Here is a packet of problems for you to ponder and work on over the summer, in order to keep your math skills fresh. Try all of the problems, and on your own paper (lined or graph paper), please write up your solutions. This means not just writing your answers, but showing your steps and your thinking! This is the work you will hand in during the first week of classes.

You may feel rusty. Each section does have worked-out examples to help you and you can check out our Summer Resources page.

We math teachers want to acknowledge that you all are coming off a challenging school year and trying to learn during the pandemic. Most people have lost class time over the past year, and we expect different people will be uncertain about different things. **So our message to you is - please just do your best!** And make note of ideas that feel unfamiliar to you. That will help us in the fall address any learning gaps you may have! We are all in this together - and we will figure it out together as well. We so appreciate your willingness to spend some time doing math this summer!

"What kind of calculator do I need to do this work?"

No calculators necessary--just use your brain! :)

"How will I get all of these problems done this summer?"

We recommend that you pace yourself throughout the summer by working through about two to three pages each week. There are optional challenges at the end.

"I don't know the answers to some of these!"

That's OK! You aren't supposed to be perfect and may not yet know how to do all of these problems. This is school, after all, and everybody's learning. Please just do your best! Check out our FAQ and Resources for resources on what to do if you feel stuck.

If you have questions about specific problems, or anything else in this packet, you can look at our FAQ and Resources page, or E-mail our department chair Joan Sullivan at jsullivan@waringschool.org.

We hope you and your family have a good summer,
The Waring Math Teaching Team

1. Evaluate each expression for the given values of the variables:

Example: $6x + 2y - z \quad for\ x = -1,\ y = 2,\ z = \frac{2}{3}$

Solution: substitute the values for the variables:

$$6 \cdot -1 + 2 \cdot 2 - \frac{2}{3} = -6 + 4 - \frac{2}{3} = -2 - \frac{2}{3} = -2\frac{2}{3}$$

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a) $5x - 4y \qquad for\ x = 9\ and\ y = 6$

b) $-6m - 2n \qquad for\ m = -\frac{1}{2}\ and\ n = \frac{1}{4}$

d) $4a^2b \qquad for\ a = -3\ and\ b = 2$

e) $2x^2 - 5x - 3 \qquad for\ x = 4$

f) $\frac{x}{3-y} \qquad for\ x = 10\ and\ y = 4$

g) $\frac{11f - 4g^4}{2} \qquad for\ f = 8\ \ and\ g = 0$

h) $\frac{1}{3}(4 - x)^2 \qquad for\ x = -5$

i) $log_2\ x \qquad for\ x = 8$

i) $b\sqrt{5 - a} \qquad for\ a = -4, b = \frac{1}{3}$

2. Rewrite the expressions as sums in simplest form, by distributing and combining like terms.

Example: $2(x + 5y) + 3(2y - 1 - 3x)$

Solution: First distribute:

$$2(x + 5y) + 3(2y - 1 - 3x) = 2x + 10y + 6y - 3 - 9x$$

Then combine like terms:

$$2x + 10y + 6y - 3 - 9x = -7x + 16y - 3$$

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a) $3n - 2 + 4n + 1$

b) $4(8y - 1) - 5y$

c) $2(a + 3b) + 9(3a + 2b)$

d) $t^2 - 59t + 54 - 82t^2 + 60t$

e) $(3x^5 + 8x^3) - (7x^2 - 6x^3)$

f) $(x^2 + 4x + 3) + 2(x^2 - x)$

g) $3n(n + 1) - 2(n^2 + 6) + n$

h) $\frac{4y^2+6y-8}{2} + y$

3. Solve the equations.

Example: $5n + 1 = 3n - 7$

Solution:

$$\begin{aligned} 5n + 1 &= 3n - 7 && \textit{subtract 3n from both sides} \\ -3n & \quad -3n \\ 2n + 1 &= -7 && \textit{subtract 1 from both sides} \\ -1 & \quad -1 \\ 2n &= -8 && \textit{divide by 2 on both sides} \\ n &= -4 \end{aligned}$$

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a) $4f + 9 = 57$

b) $-5x - 6 = 6x - 61$

c) $17 = 5 + 4(2x - 3)$

d) $15x - 3(x + 5) = 4x + 17$

e) $6(2 - 3x) + 4(1 - x) = -28$

f) $15 = \frac{x - 5}{-2}$

Each of these equations has two possible solutions:

g) $3x^2 - 12 = 0$

h) $(x - 2)(x + 7) = 0$

4. Solve the problem by writing a mathematical equation to model the situation and solve.

a) You have \$60 and your sister has \$120. You are saving \$7 per week and your sister is saving \$5 per week. How long will it be before you and your sister have the same amount of money?

Write and solve an algebraic equation for this problem.

b) You go out to dinner with your family and you pay 25% of the cost of the meal to cover tax and tips. If you have to pay \$175 in all, what was the cost of the meal before taxes and tips?

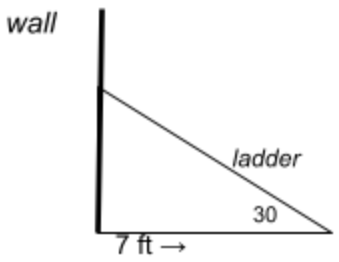
Write and solve an algebraic equation for this problem.

c) You deposit \$200 of your summer earnings into a savings account which earns 6% interest which is compounded annually. How much money will be in the account exactly one year later, if you do not deposit anything more?

d) You deposit \$200 of your summer earnings at another bank which earns 6% interest each year which is compounded monthly. How much money will be in the account after one year, if you do not deposit anything more?

e) A ladder leaning against a house makes an angle of 30° with the ground. The foot of the ladder is 7 feet from the foot of the house. How long is the ladder?

(You may use a calculator to help with this problem. Round your answer to the nearest tenth.)



5. Solve the inequality

Example: $2x + 1 \leq 9$

Solution:

$2x + 1 \leq 9$ *subtract 1 from both sides*

$2x \leq 8$ *divide by 2 on both sides*

$x \leq 4$

Next check an easy number to make sure. Let's try 0.

Substitute 0 into the original inequality: $2 \cdot 0 + 1 \leq 9$

$Is\ \ 0 + 1 \leq 9\ \ ?\ \ Yes!$ *so we want to graph to the left of 4 on the number line:*

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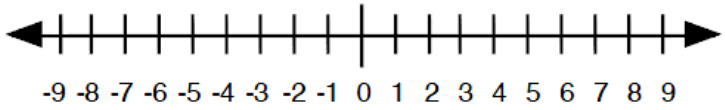
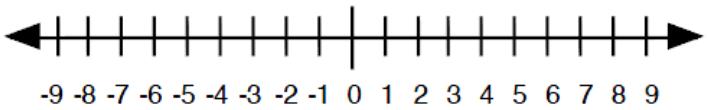
a) $x + 7 > 12$



b) $2x \leq 10$



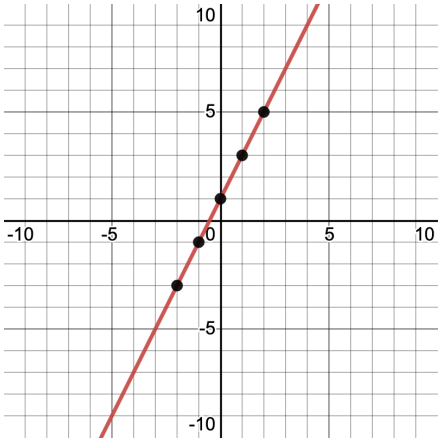
6. Complete the table of values and sketch a graph of the equation.

Example: $y = 2x + 1$

Solution: *First fill out the table by substituting values of x to find y-values.*

Then plot the points on the coordinate grid.

x	y = 2x+1
-2	**-3**
-1	**-1**
0	**1**
1	**3**
2	**4**



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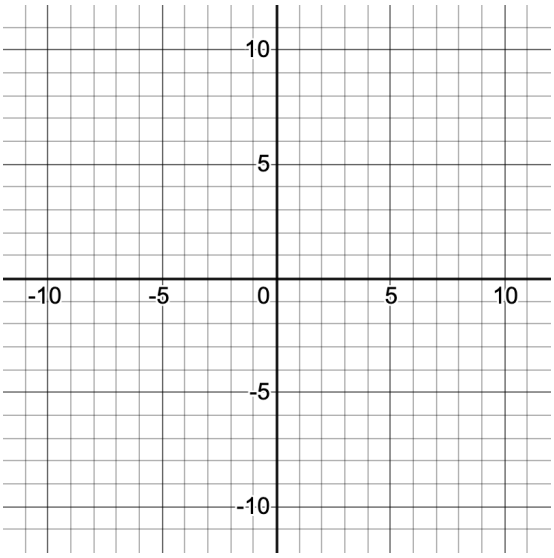
a) $y = 3x + 2$

x	y = 3 x + 2
-2	
-1	
0	
1	
2	

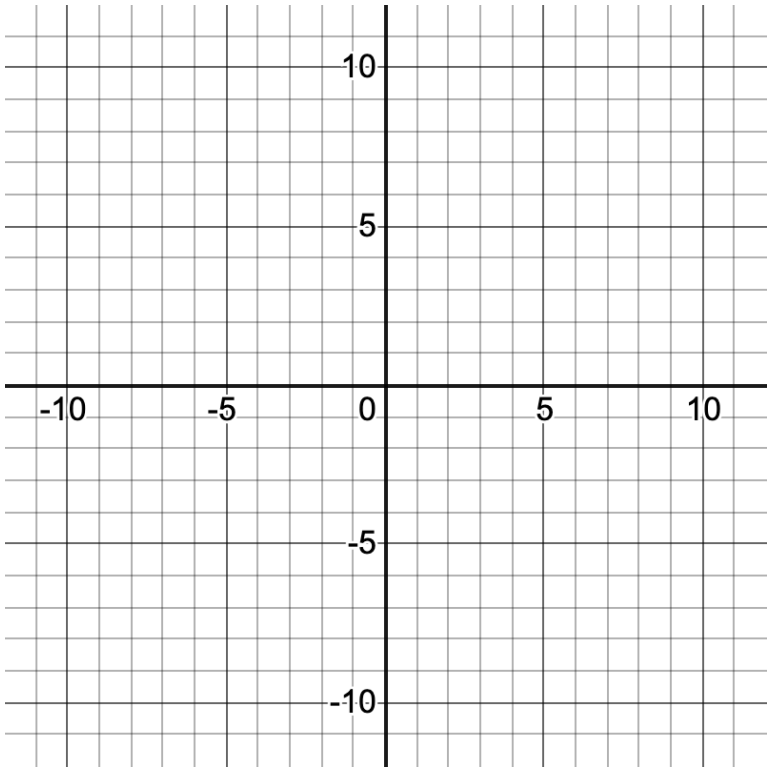


b) $y = -\frac{1}{2}x - 5$

(use the same coordinate grid for both a and b)

x	$y = -\frac{1}{2}x - 5$
-2	
0	
1	
2	

c) Graph the line that contains the point (-6, 0) and has a slope of $\frac{2}{3}$



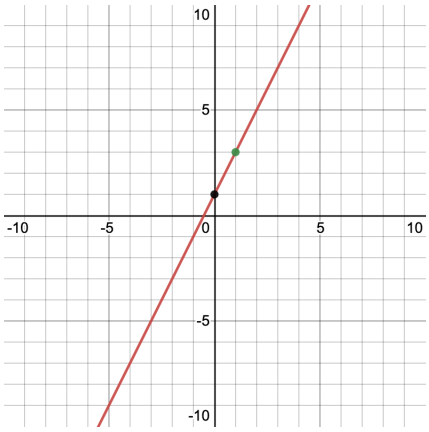
d) Graph the line that is parallel to the line in c) and includes the point (0,0).

(Use the same coordinate grid.)

What is the equation for the parallel line you just sketched?

7. Write an equation for each graph by identifying its slope and its y-intercept.

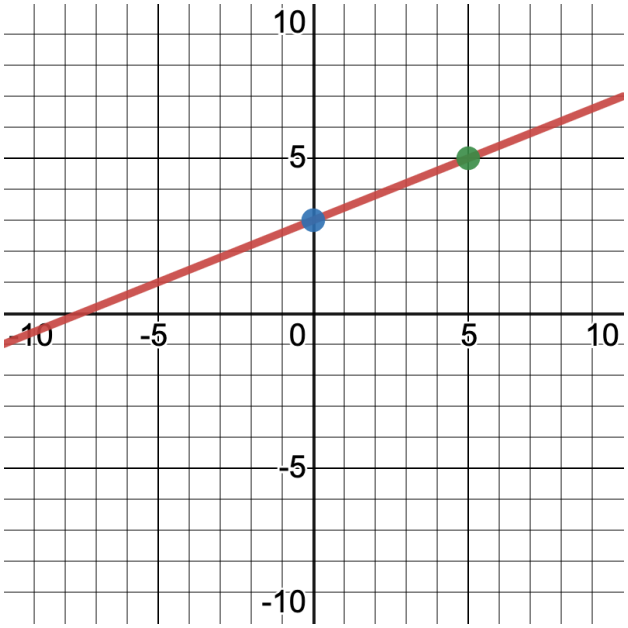
Example:



Solution: $slope = \frac{rise}{run} = \frac{2}{1} = 2$

$y - intercept = 1$

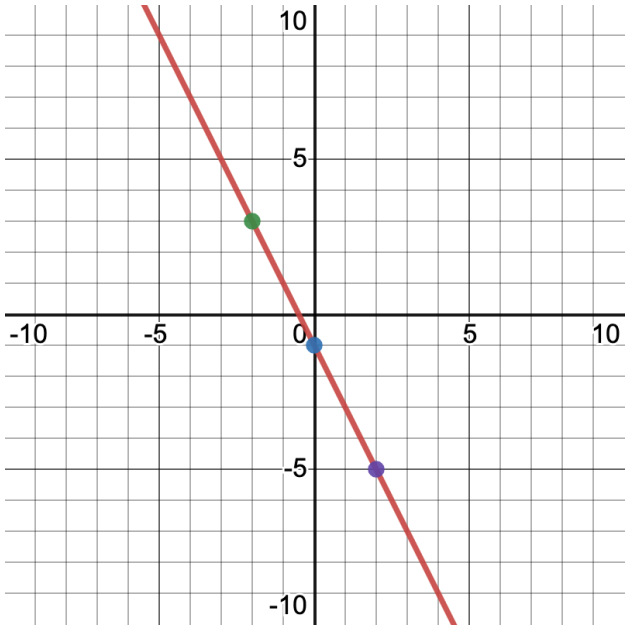
$Equation: \ y = 2x + 1$



Slope:

y-intercept :

Equation :



Slope:

y-intercept :

Equation :

8. Change the equation from standard form to slope-intercept form by solving for y.

Example: $3x + 4y = 6$

Solution: $3x + 4y = 6$ *subtract 3x from both sides*

$4y = 6 - 3x$ *divide by 4 on both sides*

$y = \frac{6-3x}{4}$ *put into slope-intercept form*

$y = \frac{6-3x}{4} = \frac{6}{4} - \frac{3}{4}x = -\frac{3}{4}x + \frac{6}{4} = -\frac{3}{4}x + \frac{3}{2}$

$y = -\frac{3}{4}x + \frac{3}{2}$

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a) -x + 4y = 8

b) 5x + 3y = 24

c) -2x - 5y = 10

9. Solve the system of equation by using substitution or elimination methods

Example: $5x + 2y = 9$
$y = -x - 3$

Solution: Using substitution we can subsitute for y in the first equation:

$5x + 2(-x - 3) = 9$	*Distribute*
$5x + -2x - 6 = 9$	*Combine like terms*
$3x - 6 = 9$	*Add 6 to both sides*
$3x = 15$	*Divide by 3 on both sides*
$x = 5$	

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a) $6y - 9 = x$
$x = -3y$

b) $3x - 2y = 4$
$5x + 2y = 12$

c) $-2x + 3y = 4$
$x = y - 3$

d) $x + y = 2$
$2x + 7y = 9$

e) $x = 2y - 4$
$7x + 5y = -66$

10. Solve the problems by writing a system of equations and solving it.

Example: There are a total of 142 laptops and desktop computers in a lab.
There are 6 more laptops than desktop computers.
What is the total number of laptops in the lab?

Solution: Let x= number of laptops and y= number of desktops
Equation 1: $x + y = 142$
Equation 2: $x = y + 6$

Using substitution:

$$(y + 6) + y = 142$$
$$2y + 6 = 142$$
$$2y = 136$$
$$y = 68$$

There are 68 desktops and 74 laptops.

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a) A company produced 300 items of clothing. Shirts were sold for $20 each, and pants were sold for $30 each. The company made total sales of $7,000 for the clothing. How many shirts did the company sell?

b) Jamie and Jenny were hungry and so they went on a food binge. Jamie bought 3 juice drinks and 2 subs for $19 and Jenny bought 2 juice drinks and 3 subs for $21. Find the price of a sub and the price of a juice drink.

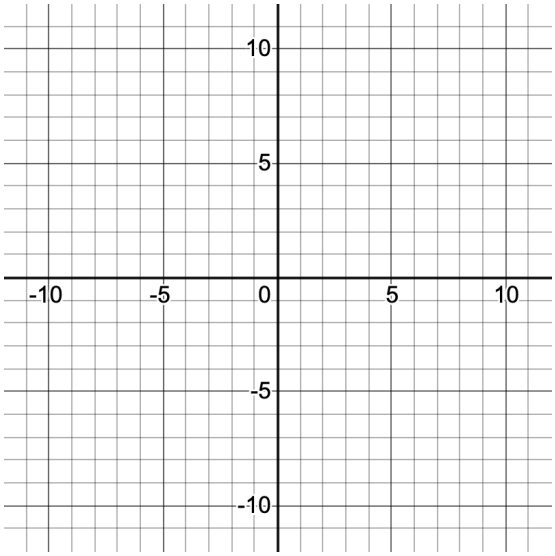
c) The difference of two numbers is 9. Their sum is 47. Find the numbers.

11. Quadratic Functions

a) Fill in the table of values and sketch a graph of the equation.

$$y = x^2 - 2x - 3$$

x	$y = x^2 - 2x - 3$
-1	
0	
1	
2	
3	

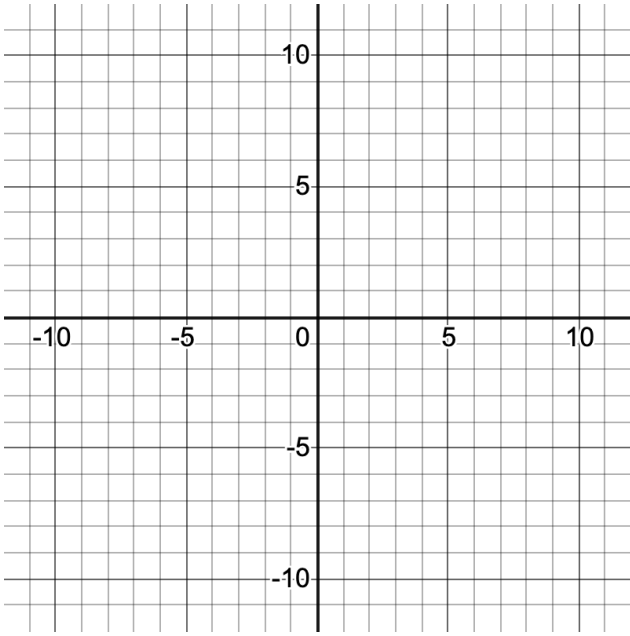


- What are the x-intercepts of the graph?
- What are the coordinates of the vertex?

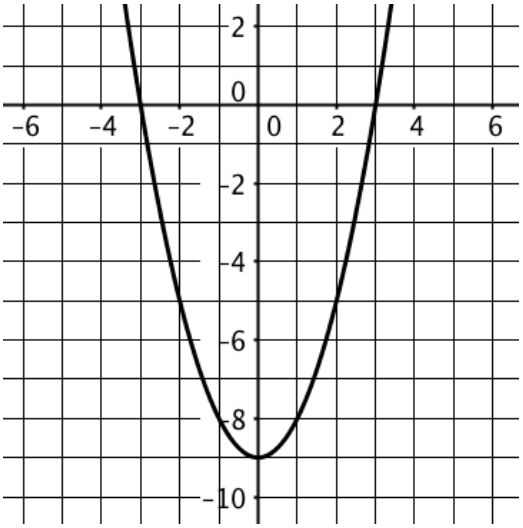
b) Sketch a graph of the equation. $y = (x + 4)(x - 2)$

- Identify the zeros.
- Identify the vertex.
- Use a table of values if that would help

x	$y = (x + 4)(x - 2)$
0	
2	



c) Here is a graph of a quadratic function, called a **parabola**:

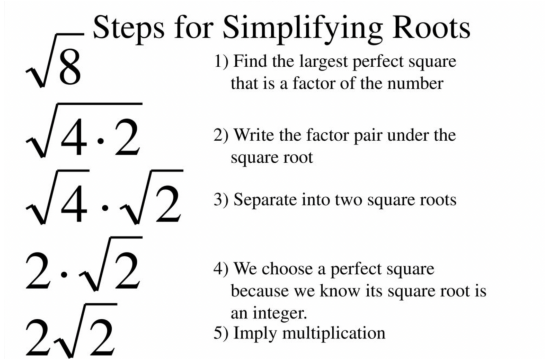


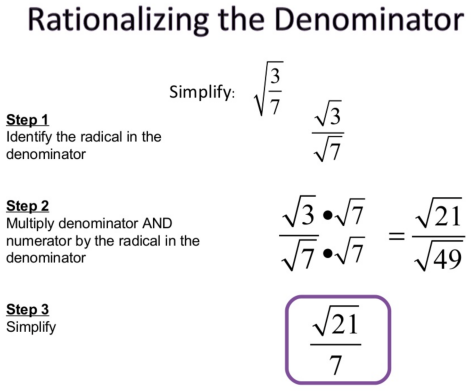
- Label the y-intercept, x-intercepts, and vertex on the parabola.

- Write an equation for this graph.

12. Radical Expressions and Fractional Powers

Examples:





$a) \sqrt{64}$

$b) \dfrac{15}{\sqrt{75}}$

$c) \sqrt[3]{64}$

$d) (27)^{1/3}$

$e) (27)^{-2/3}$

13. Functions

Given the **functions** $f(x) = x^2 - 4x \quad and \quad g(x) = 5 - x$, find each of the following:

a) $f(-2)$

b) $g(-2)$

c) $g(-x)$

d) $f(-x)$

e) Does the **set of points** {(0,1) , (1,2) , (1,4)} define a function? Why or why not?

f) Find the **domain** and **range** of the function: $y = \sqrt{x+3} - 2$

13. Logarithms and Exponents.

Simplify

$a)\ \left(2^{5a}\right)^b$

$b)\ \left(2^{3a}\right)\left(2^{5b}\right)$

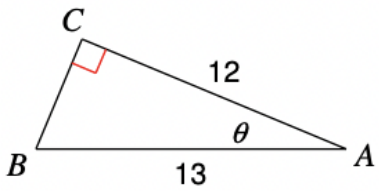
$c)\ \log 6 - \log 3$

Solve the equations.

$d) 10^{4x-1} = 1000$

$e)\ \log_2 2x + \log_2 x = 4$

14. Trigonometry and the Unit Circle



a) What is the length of side CB?
b) What is $\sin(\theta)$?
c) What is $\cos(\theta)$?
d) What is $\tan(\theta)$?

e) Sketch an angle of 120° in a **Unit Circle.** Then state the value of trig functions sine, cosine, and tangent for that angle.

This is the end ----
unless you want to try a couple more challenges (see next page)!!

Remember to bring your work to your math teacher during the first week of classes!

PROBLEMS TO PONDER : OPTIONAL CHALLENGES!

1. Use any whole numbers 1 through 9 at most one time each to create an equation whose solution would be the **largest** possible value for x.

$$\square x + \square = \square$$

2. Using the integers **-9 to 9** at most one time each, fill in the boxes to create coordinates that represent the vertices of a triangle with an area of less than 55 square units.

$$A:(\square,\square)$$
$$B:(\square,\square)$$
$$C:(\square,\square)$$

3. Using the integers 1 through 9, at most one time each, find the value of x that is closest to 0 . Extra challenge! Find more than one set of numbers that would make x = 0.

$$\log_6 \square - \log_6 \square = x$$

4. Fill in the box with any number that will create an equation with a slope of $\frac{1}{2}$.

$$3x - \square y = 12$$