Calculus AP Summer Math Packet 2021

Hi everyone!

This is a Problem Packet to review Limits and Precalculus concepts to help you get ready for the fall. Use your brain and some resources to do these problems to the best of your ability. I will collect your solutions during the first week of classes in September. On the last page you will find a list of functions, properties, and identities AP Calculus students are expected to know.

We math teachers want to acknowledge that you all are coming off a challenging school year and trying to learn during the pandemic. Most people have lost class time over the past year, and we expect different people will be uncertain about different things. **So our message to you is - please just do your best!** And make note of ideas that feel unfamiliar to you. That will help us in the fall address any learning gaps you may have! We are all in this together - and we will figure it out together as well. We so appreciate your willingness to spend some time doing math this summer!

Here is our pacing recommendation. There are about 20 problems and 7 weeks of summer, so if you tackle about 3 problems a week, then you will be all done before pre-season!

If you need a little extra help with review or the vocabulary you read here, I offer these online resources, all searchable by topic

http://www.coolmath.com/algebra (review of the basics)
https://www.coolmath.com/precalculus-review-calculus-intro (more recent material)
https://www.mathsisfun.com/algebra/index.html (great visuals for math vocabulary)
https://www.khanacademy.org/math/algebra2 (if you want video lessons)

Yes, everything in this packet is expected to be review. But you may also feel rusty when you are tackling these problems - and that's OK. Maintain your growth mindset. Chip away at these problems over the next few weeks. Take a break and come back. Check out the sites above. You got this!

If you have ANY questions or problems or just need a bit of help, please feel free to email me at jsullivan@waringschool.org. But I may be slow to respond. I'll get a reply to you as soon as I am able.

Happy summering!!!

Joan

Directions: (read carefully) Unless otherwise noted, all problems in this packet should be solved algebraically without a calculator. The standard Calculus AP exam is ⅔ non-calculator. If you need to leave an answer in logarithmic form such as x = ln 5, that's fine. I would prefer this over some rounded decimal answer anyway. If you do round, your answer should be accurate to 3 decimal places. All solutions should be written on separate sheets of paper. Make sure all solutions are numbered and easy to find. Important vocabulary has been written in **bold**.

1. Let's open with a quick Algebra "correct the mistake" activity.

True or false. If false, change what is underlined to make the statement true.

a. $(x^3)^4 = x^{\underline{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = x^{\underline{\frac{3}{2}}}$ T F

c. $(x+3)^2 = \underline{x^2+9}$ T F

d. $\dfrac{x^2-1}{x-1} = \underline{x}$ T F

e. $(4x+12)^2 = \underline{16}(x+3)^2$ T F

f. $\underline{3} + 2\sqrt{x-3} = 5\sqrt{x-3}$ T F

2. Find the **x and y-intercepts** for each of the following:

$a)\ y = x - 1$
$b)\ y = x^2 + x - 1$
$c)\ y = (x-1)\sqrt{9-x^2}$
$d)\ y = \dfrac{x-3}{(3x+1)^2}$

3. Find all **points of intersection** of each of the following:

$$a)\ \begin{aligned} 2x - 3y &= 13 \\ 5x + 3y &= 1 \end{aligned}$$

$$b)\ \begin{aligned} y &= x^3 - 4 \\ y &= -x \end{aligned}$$

4. Write the equation of the **linear function** with the following characteristics (you may use either slope-intercept form or point-slope form)

a) passes through (3, -4) and (5, 2)

b) vertical line passing through (7.-8)

c) x intercept of 5 and y intercept of -3

d) Perpendicular to the line $3x + 4y = 7$ passing through the point (-6,4)

5. SImplify the following **algebraic expressions** and rewrite without negative powers.

a) $\frac{2x^2y^{-3}}{x^3y}$

b) $-5\left(\frac{3}{2}\right)(4-9x)^{-1/2}(-9)$

c) $\frac{\frac{1}{2}(2x+5)^{-3/2}}{\frac{3}{2}}$

d) $\frac{1}{x^2} + 4x^{-2}$

6. Factor the following algebraic expressions completely. Recall: "**Factor**" means to rewrite the expression as a product. This is an important skill for Calculus, so if you are feeling rusty please check this resource: Factoring Toolkit

a) $x^2 - 4x - 5$

b) $16x^2 - 9$

c) $12x^4 + 6x^3 + 3x^2$

d) $3x^2 + 5x + 2$

(Hint: Strategy factor by grouping method.)

7. State the **zeroes** of the following functions.

a) $x^2 - 4x - 5$

b) $\frac{x^2-2x}{x^2-4}$

8. Given the **functions** $f(x) = x^2 - 4x \ \ and \ \ g(x) = 5 - x$,find each of the following:

a) $f(g(x))$

b) $g(f(x))$

c) $f(x-2)$

d) $g(2-x)$

e) $\frac{f(x)-f(3)}{x-3}$

f) $g^{-1}(x)$

9. State the **domain** and **range** of the following:

a) $f(x) = \sqrt{(x+2)}$

b) $g(x) = \frac{x+2}{x^2-4}$

10. **Graph** each of the following by hand. Use an appropriate scale and mark the x-intercept, y-intercept, and other points you think are important.

a) $y = \ -3x + 2$

b) $y = x^2 + 1$

c) $y = e^x + 1$

d) $y = \frac{x}{x-3}$

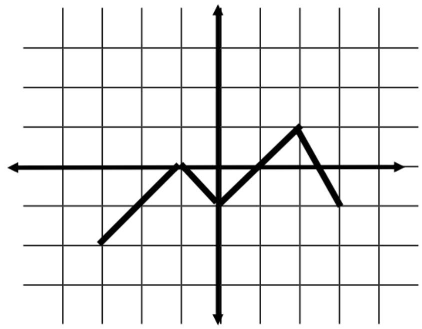
11. Solve this **quadratic equation** using the strategy called "completing the square."

$$2x^2 + 3x = 4$$

12. Given $f(x) = \frac{x^2-4}{x^2-4x^4}$, for what values of x is... ?

a) $f(x) = 0$ b) $f(x)$ **undefined**

13. The graph of function, f, is given. Graph each **transformation**.



a) $f(-x)$ b) $f(x+2)$

c) $-f(x)$ d) $f(x)+2$

14. Sketch $\frac{11\pi}{6}$ in a **Unit Circle** and state the value of all 6 trig functions.

15. State the **amplitude, period, phase shift** and **vertical shift** for the sinusoidal function $y = \cos\left(x - \frac{\pi}{3}\right) + 1$ .Then sketch one period of the function using an appropriate scale.

16. Simplify the following expression. $(\sin x)(\cos x)(\tan x)(\sec x)(\csc x)$

17. Solve the following equations. Write your answers exactly, without decimal approximations. No Calculator necessary.

a) $\sin x = \frac{1}{2},\ for\, 0 \leq x \leq 2\pi$

b) $x^2 - 3x - 4 = 0$

c) $\log_3 81 = x$

d) $\log_3 \sqrt{3} = x$

e) $2^x = 1$

f) $25e^{-x} = 50$

g) $1 = \frac{2(x+13)}{10+x}$

h) $4 = \frac{x-1}{x}$

18. ***Piecewise defined functions*** occur often in Calculus. These are functions with different rules for different pieces of their domains. Click here for more information.

$$f(x) = \begin{cases} x^2, & x < 1 \\ x - 3, & x \geq 1 \end{cases}$$

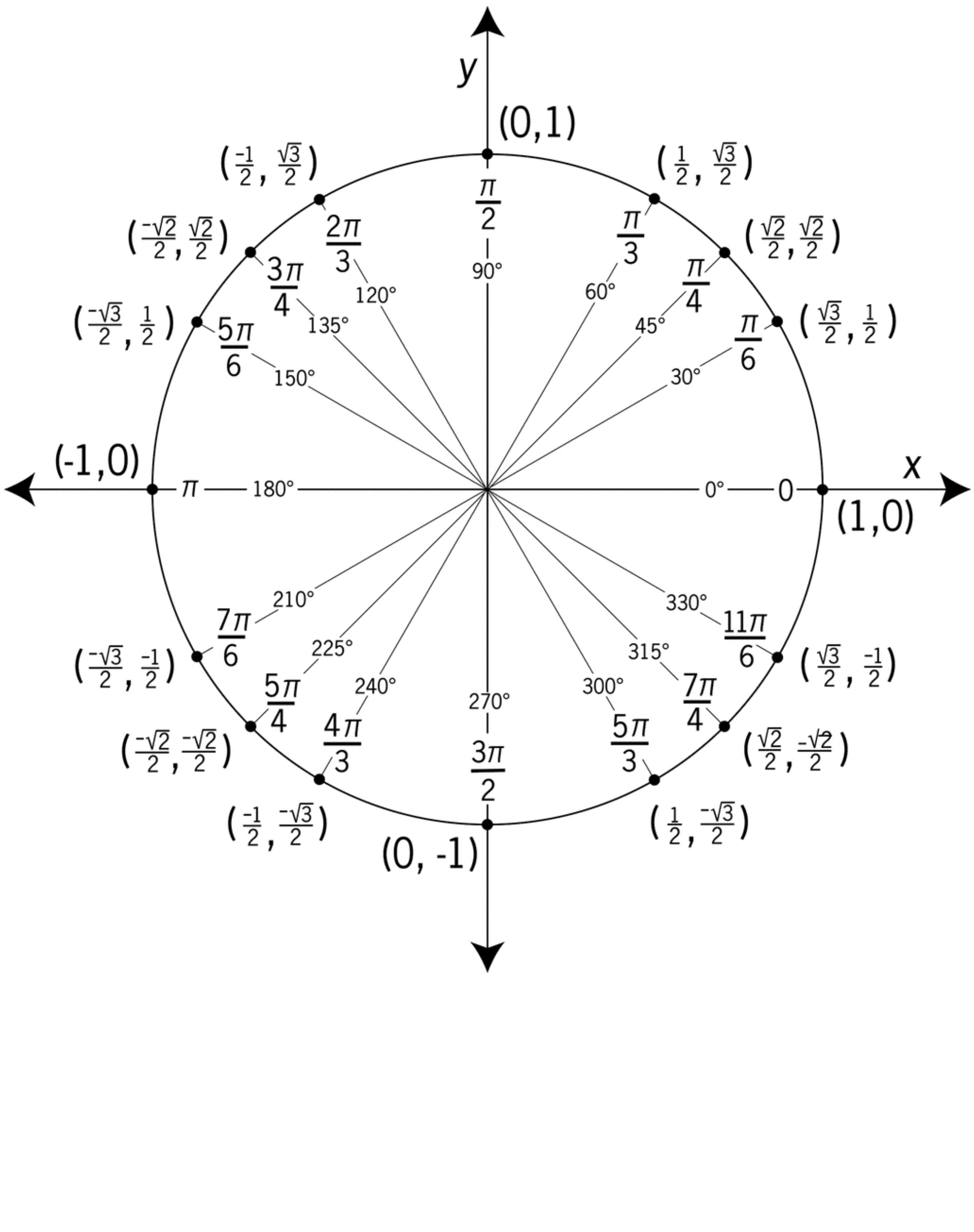
a) $f(1)$

b) $f(0)$

c) $f(-1)$

d) Make a sketch of f.

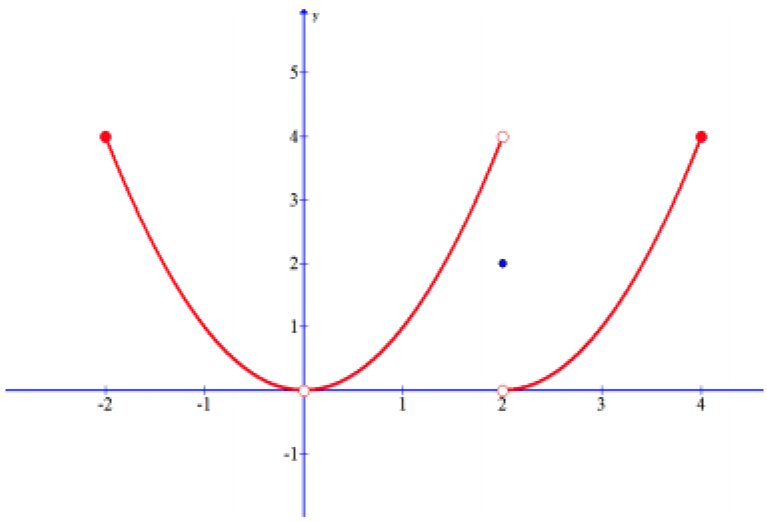
19. ***Trigonometry Facts Review***
Take time this summer to review the values of sine, cosine, and tangent in the unit circle of the "cardinal angles" in radians. You may use whatever method helps you. All the information you are expected to know is in this unit circle. Having these committed to memory will help calculus run much smoother for you. Here is a table of what you are expected to know by heart.



20. ***Limits***. In the spring you considered the idea of Limits graphically.

This is the graph of a piecewise defined function, y=f(x). Use the graph and your understanding of Limit notation to find the following:

a) $f(1)$
b) $f(2)$
c) $f(0)$
d) $\lim_{x\to 1} f(x)$
e) $\lim_{x\to 0} f(x)$
f) $\lim_{x\to 2} f(x)$
g) $\lim_{x\to 2^-} f(x)$
h) $\lim_{x\to 2^+} f(x)$
i) What is the domain of f?



...and that's the end! Thanks for making it this far!

Linear forms: Slope-intercept: $y = mx + b$ Point-slope: $y - y_1 = m(x - x_1)$

Standard: $Ax + By = C$ Horizontal line: $y = b$ (slope = 0)

Vertical line: $x = a$ (slope is undefined)

Parallel → Equal slopes Perpendicular → Slopes are opposite reciprocals

Quadratic forms: $y = ax^2 + bx + c$ $y = a(x-h)^2 + k$ $y = a(x-p)(x-q)$

Reciprocal Identities: $\csc x = \frac{1}{\sin x}$ $\sec x = \frac{1}{\cos x}$ $\cot x = \frac{1}{\tan x}$

Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$ $\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin(2x) = 2\sin x \cos x$ $\tan(2x) = \frac{2\tan x}{1-\tan^2 x}$

$$\cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Exponential Properties: $x^a \cdot x^b = x^{a+b}$ $(xy)^a = x^a y^a$ $x^0 = 1$ for all $x \neq 0$

$\frac{x^a}{x^b} = x^{a-b}$ $\left(\frac{x}{y}\right)^a = \frac{x^a}{x^b}$ $\sqrt[b]{x^n} = x^{n/b}$ $x^{-n} = \frac{1}{x^n}$

Logarithms: $y = \log_a x$ is equivalent to $a^y = x$

Logarithmic Properties: $\log_b mn = \log_b m + \log_b n$ $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$

$\log_b(m^p) = p \cdot \log_b m$ If $\log_b m = \log_b n$, then $m = n$ $\log_a n = \frac{\log_b n}{\log_b a}$

Note:
You should be confident in manipulating rational expressions and factoring algebraic expressions. The pythagorean and double angle identities we will discuss in class in September.